

# Boarshaw Primary School

## Calculation Policy

**Adopted: February 2020**

Original document written and prepared by Anthony Reddy, Dave Godfrey and Laurence Hicks

Sense of Number Maths Consultants

© [www.senseofnumber.co.uk](http://www.senseofnumber.co.uk)



### Contents:

Page

**2: General Principles of Calculation**

**3: Concrete & Pictorial Resources / 'Mastery' of Mathematics**

**4: Overview of Calculation Approaches**

**8: Calculation Vocabulary**

**9: Mental Methods of Calculation**

**10: CPA & Informal Written Methods / Mental Jottings**

**11: Formal (Column) Written Methods of Calculation**

**12: National Curriculum Objectives – Addition and Subtraction**

**13: National Curriculum Objectives – Multiplication and Division**

**14: Addition Progression**

**26: Subtraction Progression**

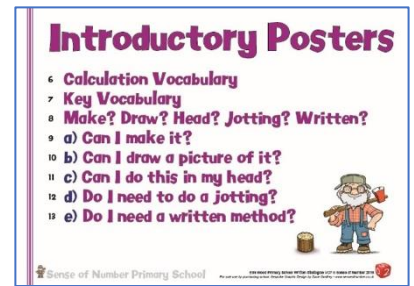
**40: Multiplication Progression**

**60: Division Progression**



# General Principles of Calculation

When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. Whatever method is chosen (in any year group), it must still be underpinned by a secure and appropriate knowledge of number facts.



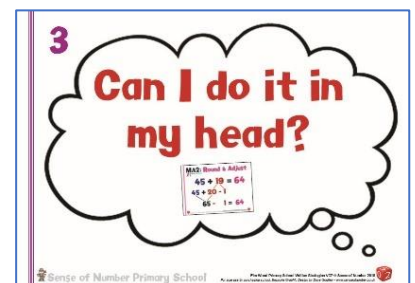
By the end of Year 5, children should:

- have a secure knowledge of number facts and a good understanding of the four operations in order to:
  - carry out calculations mentally when using one-digit and two-digit numbers
  - use particular strategies with larger numbers when appropriate
- use notes and jottings to record steps and part answers when using longer mental methods
- **have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;**

Children should always **look at the actual numbers (not the size of the numbers)** before attempting any calculation to determine whether or not they need to use a written method.

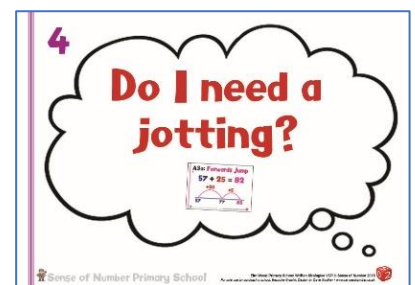
Therefore, the key question children should always ask themselves before attempting a calculation is

**'Can I do this in my head?'**

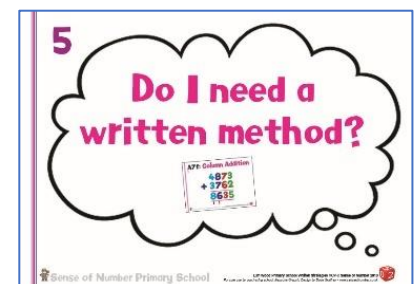


This should be the first principle for any calculation, whether addition, subtraction, multiplication & division. If a child is asking themselves this question it means that they are looking at the actual numbers and using their sense of number to determine whether there is an easier method that they could use rather than column / standard procedures.

In the previous curriculum, the next question that children should ask themselves (if they could answer the calculation mentally) would be whether or not they needed to jot something down to support their thinking and help them calculate – **'Do I need a jotting?'**. If the answer to this question was **'yes'** then the children would access the wide range of mental strategies & jottings outlined later in this document.



If, however, the numbers were too difficult or unwieldy for a mental method or jotting to be appropriate then they would ask the question **'Do I need a written method?'**. Again, if the written method was the most efficient and appropriate way to find the answer, they would access one of the written methods (either informal or standard) found later in this document.



**(NB. Section 5 explains this principle in detail, with examples for each of the four operations)''''**

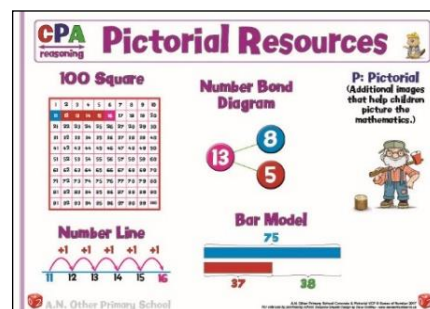
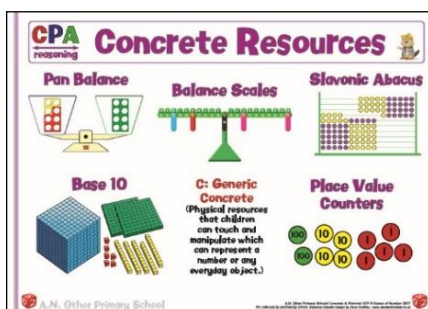
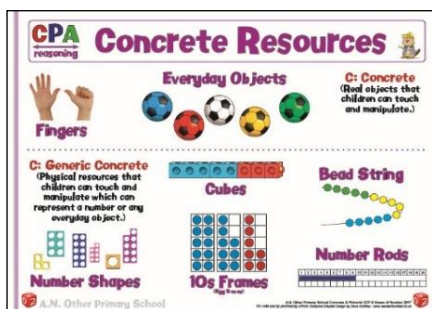
The 2014 National Curriculum, though, recommends a **Mastery** approach to teaching calculation, meaning that there are now 2 additional questions which children should be asking before moving to jottings or written methods – ‘**Can I make it?**’ and ‘**Can I draw a picture of it?**’.

These are explained in the next section. (see below)

## Concrete and Pictorial Resources / ‘Mastery’ of Mathematics

The ability of a child to work calculations out mentally, and also to successfully master a written procedure should initially come from a secure understanding of each calculation.

This is achieved through the use of concrete and pictorial resources throughout EYFS and both Key Stages.



The 2014 National Curriculum advocates a ‘**mastery**’ approach to mathematics suggesting all children should master conceptual understanding of any calculation they are attempting to solve.

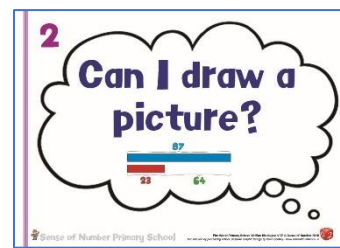
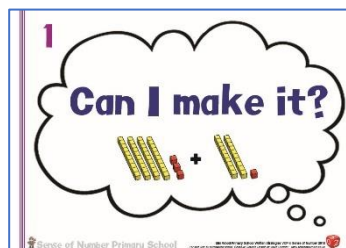
**Mastery** is the approach adopted in the Far East when teaching mathematics, (particularly in Singapore and Shanghai), and aims to ensure that children are given a substantial depth of understanding in place value for each year group.

This understanding, which is built up through regular use of concrete apparatus becomes the foundation of the **CPA (Concrete – Pictorial – Abstract)** approach to teaching.

Children who have developed secure visual place value through the use of concrete apparatus, manipulatives and images (such as **Base 10 / Tens Frames / Number Rods / Numicon / Place Value Counters** etc) are then able to apply it when learning methods of calculation.

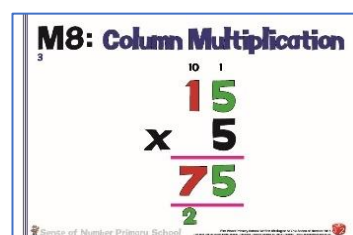
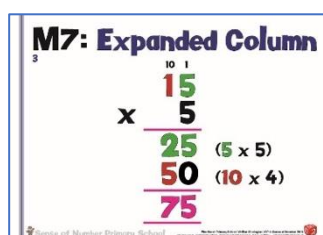
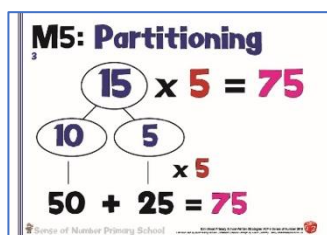
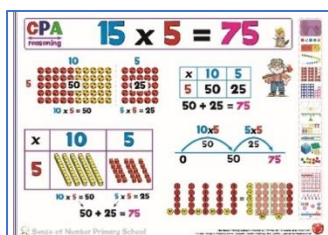
This ensures that they fully understand a calculation rather than just learning a method by rote. Consequently, the written method ‘makes sense’ and can be retained much more easily as it is based on conceptual understanding.

Therefore, a child who has ‘mastered’ a calculation can now ask themselves two additional questions that enable them to either explain their understanding (‘**Can I make it?**’) or give them a visual alternative to a written method (‘**Can I draw a picture?**’).



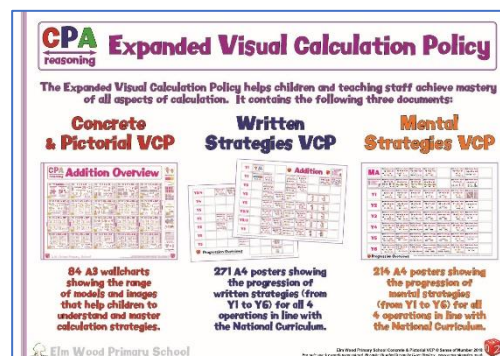


The example below shows the development of  $15 \times 5$  from **Concrete** understanding (**Place Value Counters and Base 10**) to **mental jottings** (**Number Line**) to **informal methods** that support mental arithmetic (**Grid Method / Partitioning**) to an **expanded method** and finally to the **column procedure**.



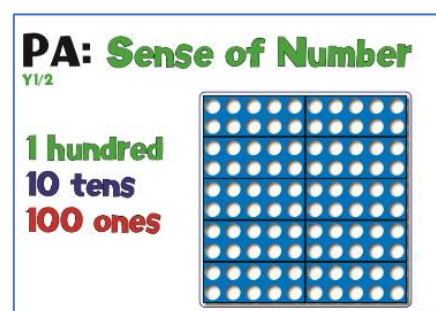
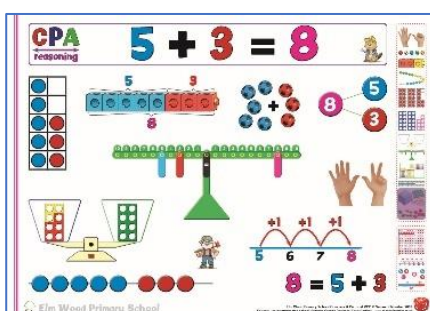
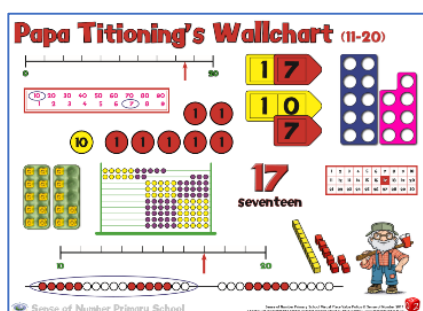
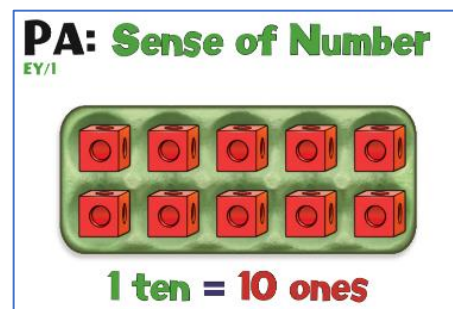
This calculation policy has been upgraded in line with the 2014 National Curriculum to reflect Mastery, and (as seen above) each calculation method now begins with Concrete materials before progressing to the mental and written methods advocated by the previous curriculum.

## Overview of Calculation Approaches



### Early Years into KS1

- Visualisation to secure understanding of the number system, especially the use of place value resources such as Tens Frames, Base 10, Numicon, 100 Squares and abacuses.
- Secure understanding of numbers to 10, using resources such as Tens Frames / Egg Boxes, Numicon, fingers, tallies and multi-link.
- Subitising to begin making links between the different images of a number and their links to calculation.
- Practical, oral and mental activities to understand calculation
- Personal methods of recording.





# Key Stage 1

- Introduce signs and symbols ( $+$ ,  $-$ ,  $\times$ ,  $\div$  *in Year 1* and  $<$ ,  $>$  *signs in Year 2*)
- Extended visualisation to secure understanding of the number system beyond 100, especially the use of place value resources such as Base 10, Place Value Charts & Grids, Number Grids, Arrow Cards and Place Value Counters.
- Further work on subitising and Tens Frames to develop basic calculation understanding, supported by Numicon and multi-link.
- Continued use of practical apparatus to support the early teaching of 2-digit calculation. For example, using Base 10 or Numicon to demonstrate partitioning and exchanging before these methods are taught as jottings / number sentences.
- Methods of recording / jottings to support calculation (e.g. partitioning or counting on).
- Use images such as empty number lines to support mental and informal calculation. Children start to adopt a range of mental strategies that they apply when appropriate, especially partitioning for addition and counting on for subtraction.

**CPA reasoning**  $8 + 5 = 13$

Elm Wood Primary School

**CPA reasoning**  $57 + 25 = 82$

Elm Wood Primary School

**CPA reasoning**  $75 - 37 = 38$

Elm Wood Primary School

**MA3: Partitioning**

$57 + 25 = 82$

$70 + 12 = 82$

Sense of Number Primary School

**A2a: Counting On**

Bridging 10

$8 + 5 = 13$

Sense of Number Primary School



## Years 3 & 4

- Continued use of practical apparatus, especially Place Value Counters, Base 10 and Numicon to visualise written / column methods before and as they are actually taught as procedures.

Children need to be given ample opportunity within their maths lessons to 'make' the calculations, dealing with the principles of exchanging / regrouping in Ones, Tens & Hundreds.

Once they can make and explain a procedure with apparatus then they can firstly draw the calculation and finally use column procedures.

- Continued use of mental methods and jottings for 2 and 3-digit calculations. As before, the first principle is still **'Can I do it in my head?'**
- Introduction to more efficient informal written methods / jottings including expanded methods and efficient use of number lines (especially for subtraction).
- Column methods, where appropriate, for 3-4-digit additions and subtractions.

**CPA Reasoning**  $723 - 356 = 367$  Part 1

Sense of Number Primary School

**S8a: Part/Whole (S)** Partition the Subtrahend

$75 - 37 = 38$

$75 - 35 = 40$  |  $40 - 2 = 38$

Sense of Number Primary School

**MA1: Manipulate Calculation**  $4645 + 1996 = 6641$

$4641 + 2000 = 6641$

Sense of Number Primary School

**S10: Expanded Column** Subtraction (100, 10, 1s)

$723 - 356 = 367$

600	110	1
<del>700</del>	<del>20</del>	3
- 300	50	6
300	60	7

Sense of Number Primary School

**A8f: Column Addition**

$4873 + 3762 = 8635$

Sense of Number Primary School

**M8b: Column Multiplication**

$147 \times 4 = 588$

Sense of Number Primary School

## Years 5-6

- Even in Years 5 & 6 it is crucial that the children have a visual picture of certain calculations in order to 'master' their understanding of what the calculation actually looks like.

They must still be allowed access to **practical resources** to help visualise these calculations, including those involving decimals

For 5/6-digit numbers this is usually quite difficult, and therefore the children will instead need to explain the methods using place value.

- Continued use of mental methods for any appropriate calculation up to 6 digits.

Even when the numbers are much larger or contain decimals, children need to use their sense of number to determine when a mental method or jotting would be far more appropriate than a written procedure.

This will include all four operations and involves the children learning and applying a detailed bank of mental strategies depending on the size and value of the numbers in question.

- Standard written (compact) / column procedures to be learned for all four operations
- Efficient informal methods (expanded addition and subtraction, grid multiplication, division by chunking) and number lines are still used when appropriate. Develop these to larger numbers and decimals where appropriate.

**CPA Reasoning**  $4.8 + 3.8 = 8.6$

1. Base Ten Blocks

2. Number Line

3. Bar Model

Elm Wood Primary School

**MA1: Manipulate Calculation**

$45.2 + 49.9 = 95.1$

$45.1 + 50 = 95.1$

Sense of Number Primary School

**MM1b: Manipulate Calculation**

$45 \times 14$

$90 \times 7 = 630$

Sense of Number Primary School

**S11h: Column Subtraction**

$742831 - 427358 = 315473$

Sense of Number Primary School

**S7i: Decimal T-J!**

$13.4 - 8.7 = 4.7$

Sense of Number Primary School



# The Importance of Vocabulary in Calculation

It is vitally important that children are exposed to the relevant calculation vocabulary throughout their progression through the four operations.

**Key Vocabulary:** (to be used from Y1)

**Addition:** **Total & Sum** **Add**

E.g. 'The **sum** of 12 and 4 is 16', '12 **add** 4 equals 16'  
'12 and 4 have a **total** of 16'

**Subtraction:** **Difference**

**Subtract** (not 'take away' unless the strategy is take away / count back)

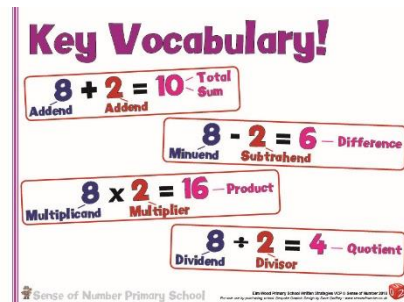
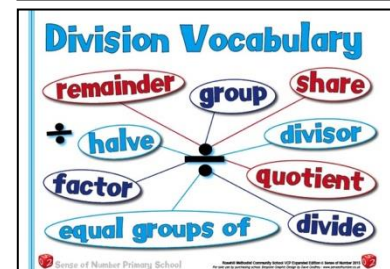
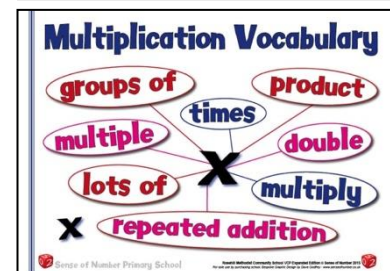
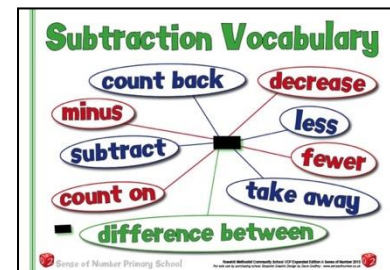
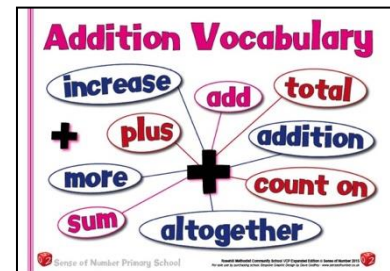
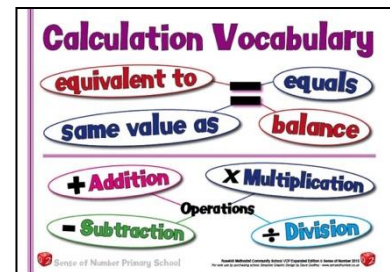
E.g. 'The **difference** between 12 and 4 is 8',  
'12 **subtract** 4 equals 8'

**Multiplication:** **Product** **Multiply**

E.g. 'The **product** of 12 and 4 is 48',  
'12 **multiplied** by 4 equals 48'

**Division:** **Divisor & Quotient** **Divide**

E.g. 'The **quotient** of 12 and 4 is 3',  
'12 **divided** by 4 equals 3'  
'When we **divide** 12 by 4, the **divisor** of 4 goes into 12 three times'

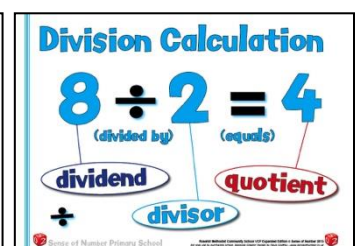
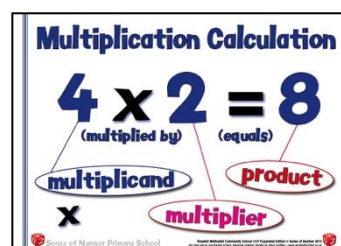
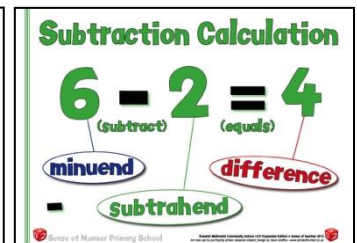
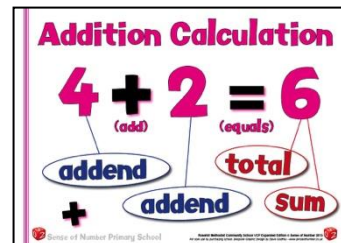


**Additional Vocabulary:**

The VCP vocabulary posters contain both the key and additional vocabulary children should be exposed to.

**Conceptual Understanding**

Using key vocabulary highlights some important conceptual understanding in calculation. For example, the answer in a subtraction calculation is called the difference. Therefore, whether we are counting back (taking away), or counting on, to work out a subtraction calculation, either way we are always finding the difference between two numbers.



## Mental Methods of Calculation

Oral and mental work in mathematics is essential, particularly so in calculation.

Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts.

Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied.

On-going oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a **'sense' of number** is the product of structured practice and repetition. It requires an understanding of **number patterns and relationships** developed through **directed enquiry**, use of **models and images** and the application of **acquired number knowledge and skills**. Secure mental calculation requires the ability to:

- **recall key number facts instantly** – for example, all **number bonds to 20**, and **doubles** of all numbers up to **double 20 (Year 2)** and **multiplication facts up to  $12 \times 12$  (Year 4)**;
- **use taught strategies to work out the calculation** – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number to a one-digit or two-digit number (**Year 1**), add two-digit numbers in different ways (**Year 2**), add and subtract numbers mentally with increasingly large numbers (**Year 5**);
- understand how the rules and laws of arithmetic are used and applied – for example to use **commutativity** in multiplication (**Year 2**), **estimate** the answer to a calculation and use **inverse operations** to check answers (**Years 3 & 4**), use their knowledge of the **order of operations** to carry out calculations involving the four operations (**Year 6**).

The first 'answer' that a child may give to a mental calculation question would be based on instant recall.

E.g. "What is  $12 + 4$ ?", "What is  $12 \times 4$ ?", "What is  $12 - 4$ ?" or "What is  $12 \div 4$ ?" giving the immediate answers "16", "48", "8" or "3"

Other children would still work these calculations out mentally by counting on from 12 to 16, counting in 4s to 48, counting back in ones to 8 or counting up in 4s to 12.

From instant recall, children then develop a bank of mental calculation strategies for all four operations, in particular addition and multiplication.

These would be practised regularly until they become refined, where children will then start to see and use them as soon as they are faced with a calculation that can be done mentally.

**MA4: Double & Adjust**

$$45 + 46 = 91$$
$$45 + 45 + 1$$
$$90 + 1 = 91$$

Sense of Number Primary School

# CPA & Informal Written Methods / Mental Jottings

The *New Curriculum for Mathematics* sets out progression in written methods of calculation, which highlights the compact written methods for each of the four operations. It also places emphasis on the need to ‘*add and subtract numbers mentally*’ (Years 2 & 3), mental arithmetic ‘*with increasingly large numbers*’ (Years 4 & 5) and ‘*mental calculations with mixed operations and large numbers*’ (Year 6).

There is very little guidance, however, on the ‘jottings’ and informal methods that support mental calculation, and which provide the link between answering a calculation entirely mentally (without anything written down) and completing a formal written method with larger numbers.

Although the New Curriculum, as mentioned previously, also advocates the Mastery / CPA (Concrete – Pictorial – Abstract) approach to teaching calculation, it again gives almost no guidance as to what this approach would look like in the classroom.

This policy, for all four operations, provides very clear guidance not only as to the development of formal written methods, but also: -

- some of the key concrete materials and apparatus that can be used in the classroom to support visualisation and depth of understanding for many of the calculation methods
- the jottings, expanded & informal methods of calculation that embed a sense of number and understanding before column methods are taught. These valuable strategies include:

**Addition** – concrete materials      number lines      partitioning      expanded methods

The first poster shows CPA concrete materials for  $57 + 25 = 82$ . The second, A3a Forwards Jump, shows a number line from 57 to 82 with jumps of +20 and +5. The third, A5a Partition Jot, shows  $57 + 25 = 82$  partitioned into  $70 + 12 = 82$ . The fourth, A6 Expanded Column, shows the expanded column method for  $57 + 25 = 82$ .

(In addition to the 6 key mental strategies for addition - see ‘[Addition Progression](#)’)

**Subtraction** – concrete materials      number lines (especially for counting on)      expanded subtraction

The first poster shows CPA concrete materials for  $75 - 37 = 38$ . The second, S7 Backwards Jump, shows a number line from 75 to 38 with jumps of -30 and -7. The third, S8 Triple Jump!, shows a number line from 37 to 75 with jumps of +3, +30, and +5. The fourth, S10 Expanded Column, shows the expanded column method for  $723 - 356 = 367$ .

(In addition to the 6 key mental strategies for subtraction - see ‘[Addition Progression](#)’)

## Multiplication

Concrete materials      number lines      partitioning      expanded      grid method

The first poster shows CPA concrete materials for  $15 \times 5 = 75$ . The second, M4 Multi Boing!, shows a number line from 0 to 75 with jumps of  $10 \times 5$  and  $5 \times 5$ . The third, M5 Partitioning, shows  $15 \times 5 = 75$  partitioned into  $10 \times 5 = 50$  and  $5 \times 5 = 25$ . The fourth, M6 Expanded Column, shows the expanded column method for  $15 \times 5 = 75$ . The fifth, M5 Grid Method, shows a grid for  $15 \times 5 = 75$ .

(In addition to the 10 key mental strategies for multiplication (see ‘[Multiplication Progression](#)’)

**Division**      Concrete materials      number lines      chunking      formal method

The first poster shows CPA concrete materials for  $72 \div 4 = 18$ . The second, D7 Chunking Jump, shows a number line from 0 to 72 with jumps of  $4 \times 10$  and  $4 \times 8$ . The third, D6 Find the Hunk!, shows  $72 \div 4 = 18$  partitioned into  $40 \div 4 = 10$  and  $32 \div 4 = 8$ . The fourth, D10b Short Division, shows the short division method for  $136 \div 4 = 34$ .

(In addition to the 7 key mental strategies for division (see ‘[Division Progression](#)’)





## Formal (Column) Written Methods of Calculation

The aim is that by the end of **Year 5**, the great majority of children should be able to **use an efficient written method for each operation with confidence and understanding with up to 4 digits**.

This guidance promotes the use of what are commonly known as 'standard' written methods – methods that are efficient and work for any calculation, including those that involve whole numbers or decimals. They are compact & consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that schools adopt greater consistency in their approach to calculation that all teachers understand and towards which they work.

There has been some confusion previously in the progression towards written methods and for too many children the staging posts along the way to the more compact method have instead become end points. While this may represent a significant achievement for some children, the great majority are entitled to learn how to use the most efficient methods.

**The challenge for teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.**

The incidence of children moving between schools and localities is very high in some parts of the country. Moving to a school where the written method of calculation is unfamiliar and does not relate to that used in the previous school can slow the progress a child makes in mathematics. There will be differences in practices and approaches, which can be beneficial to children. **However, if the long-term aim is shared across all schools and if expectations are consistent then children's progress will be enhanced rather than limited.**

The entitlement to be taught how to use efficient written methods of calculation is set out clearly in the National Curriculum objectives. Children should be equipped to decide when it is best to use a mental or written method based on the knowledge that they are in control of this choice as they are able to carry out all methods with confidence.

This policy does, however, clearly recognise that whilst children should be taught the efficient, formal written calculation strategies, **it is vital that they have exposure to models and images (CPA), and have a clear conceptual understanding of each operation and each strategy.**

The visual slides that feature below (in the separate progression documents) for all four operations have been taken from the Sense of Number Visual Calculations Policy.

They show, wherever possible, the different strategies for calculation exemplified with identical values. This allows children to compare different strategies and to ask key questions, such as, **'what's the same, what's different?'**

The image displays four different methods for calculating  $147 \times 4 = 588$ :

- CPA reasoning:** Shows the calculation using base ten blocks. 147 is represented by 1 hundred block, 4 ten rods, and 7 one units. Multiplying by 4 gives 4 hundreds, 16 tens, and 28 ones, which are then regrouped to 400 + 160 + 28 = 588.
- M5b: Grid Method:** A grid with 100, 40, and 7 in the top row and 4 in the left column. The products are 400, 160, and 28. The total is  $400 + 160 + 28 = 588$ .
- M6: Expanded Column:** Shows the expanded column method:  $147 \times 4 = 588$ . The steps are:  $7 \times 4 = 28$  (4 x 7),  $40 \times 4 = 160$  (4 x 40), and  $100 \times 4 = 400$  (4 x 100). The final sum is 588.
- M7: Column Multiplication:** Shows the standard column multiplication:  $147 \times 4 = 588$ . The steps are:  $7 \times 4 = 28$  (write 8, carry 2),  $40 \times 4 = 160$  (write 0, carry 16), and  $100 \times 4 = 400$  (write 00, carry 4). The final sum is 588.



# National Curriculum Objectives – Addition and Subtraction

## Addition & Subtraction

1

2

3

4

5

6

<p><b>Problem Solving</b></p> <p>1 solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as <math>7 = [ ] - 9</math>.</p> <p>2 represent and use number bonds and related subtraction facts within 20</p> <p>3 recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100</p> <p>4 solve problems with addition and subtraction:  ***Using concrete objects and pictorial representations, including those involving numbers, quantities and measures  applying their increasing knowledge of mental and written methods</p> <p>5 solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.</p> <p>6 solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why  solve problems involving addition, subtraction, multiplication and division</p>	<p><b>Facts</b></p> <p>1 read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs</p> <p>2 add and subtract one-digit and two-digit numbers to 20, including zero</p> <p>3 estimate the answer to a calculation and use inverse operations to check answers</p> <p>4 estimate and use inverse operations to check answers to a calculation</p> <p>5 use rounding to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy</p> <p>6 use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy  use their knowledge of the order of operations to carry out calculations involving the four operations</p>	<p><b>Understanding and Using Statements &amp; Relationships</b></p> <p>1 add and subtract numbers *using concrete objects, pictorial representations, and mentally, including:  a two-digit number &amp; ones  a two-digit number &amp; tens  two two-digit numbers  adding three one-digit numbers</p> <p>2 add and subtract numbers mentally, including:  a three-digit number &amp; ones  a three-digit number &amp; tens  a three-digit number and hundreds</p> <p>3 add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction</p> <p>4 add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)  add and subtract numbers mentally with increasingly large numbers</p> <p>5 perform mental calculations, including with mixed operations and large numbers</p>	<p><b>Addition and Subtraction – Mental &amp; Written Methods</b></p> <p>1 Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, <math>9 + 7 = 16</math>; <math>16 - 7 = 9</math>; <math>7 = 16 - 9</math>). They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations.  Pupils combine and increase numbers, counting forwards and backwards.  They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.</p> <p>2 Pupils extend their understanding of the language of addition and subtraction to include sum and difference.  Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using <math>3 + 7 = 10</math>; <math>10 - 7 = 3</math> and <math>7 = 10 - 3</math> to calculate <math>30 + 70 = 100</math>; <math>100 - 70 = 30</math> and <math>70 = 100 - 30</math>. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, <math>5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5</math>). This establishes commutativity and associativity of addition.  Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers</p> <p>3 Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100.  Pupils use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent (see <a href="#">Mathematics Appendix 1</a>).</p> <p>4 Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency (see <a href="#">English Appendix 1</a>).</p> <p>5 Pupils practise using the formal written methods of columnar addition and subtraction to aid fluency (see <a href="#">Mathematics Appendix 1</a>).  They undertake mental calculations with increasingly large numbers to aid fluency (for example, <math>12\ 462 - 2300 = 10\ 162</math>).</p> <p>6 Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see <a href="#">Mathematics Appendix 1</a>).  They undertake mental calculations with increasingly large numbers and more complex calculations.  Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.  Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.  Pupils explore the order of operations using brackets; for example, <math>2 + 1 \times 3 = 5</math> and <math>(2 + 1) \times 3 = 9</math>.  Common factors can be related to finding equivalent fractions.</p>
--	--	--	---

# National Curriculum Objectives – Multiplication and Division

1	2	3	4	5	6
<h2>Multiplication &amp; Division</h2>					
<h3>Problem Solving</h3>					
<ul style="list-style-type: none"> <li>solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</li> </ul>	<ul style="list-style-type: none"> <li>solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.</li> </ul>	<ul style="list-style-type: none"> <li>solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.</li> </ul>	<ul style="list-style-type: none"> <li>solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.</li> </ul>	<ul style="list-style-type: none"> <li>solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes</li> <li>solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign</li> <li>solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.</li> </ul>	<ul style="list-style-type: none"> <li>solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why</li> <li>solve problems involving addition, subtraction, multiplication and division</li> <li>use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.</li> </ul>
<h3>Facts</h3>					
<ul style="list-style-type: none"> <li>recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers</li> </ul>	<ul style="list-style-type: none"> <li>recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables</li> </ul>	<ul style="list-style-type: none"> <li>recall multiplication and division facts for multiplication tables up to <math>12 \times 12</math></li> </ul>	<ul style="list-style-type: none"> <li>establish whether a number up to 100 is prime and recall prime numbers up to 19</li> </ul>	<ul style="list-style-type: none"> <li>identify common factors, common multiples and prime numbers</li> <li>use their knowledge of the order of operations to carry out calculations involving the four operations</li> </ul>	
<h3>Understanding and Using Statements &amp; Relationships</h3>					
<ul style="list-style-type: none"> <li>show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot</li> </ul>	<ul style="list-style-type: none"> <li>use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers</li> <li>recognise and use factor pairs and commutativity in mental calculations</li> <li>recognise and use square numbers and cube numbers, and the notation for squared (<math>^2</math>) and cubed (<math>^3</math>)</li> </ul>	<ul style="list-style-type: none"> <li>write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers; using mental and progressing to formal written methods</li> </ul>	<ul style="list-style-type: none"> <li>multiply two-digit and three-digit numbers by a one-digit number using formal written layout</li> <li>multiply and divide numbers mentally drawing upon known facts</li> <li>divide numbers up to 4 digits by a one-digit number using the formal written method of short division and those involving decimals by 10, 100 and 1000</li> </ul>	<ul style="list-style-type: none"> <li>multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</li> <li>divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</li> <li>divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context</li> <li>perform mental calculations, including with mixed operations and large numbers</li> </ul>	
<h3>Multiplication and Division – Mental &amp; Written Methods</h3>					

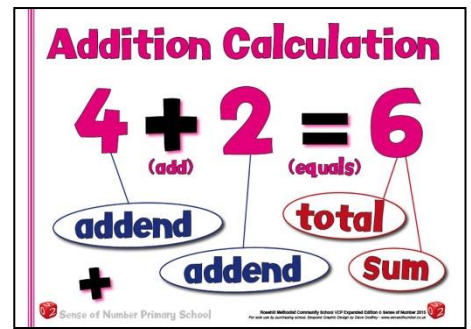




# Addition Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Children need to acquire **one efficient written method of calculation** for addition that they know they can rely on **when mental methods are not appropriate**.



To add successfully, children need to be able to:

- recall all addition pairs to  $9 + 9$  and complements in 10;
- add mentally a series of one-digit numbers, such as  $5 + 8 + 4$ ;
- add multiples of 10 (such as  $60 + 70$ ) or of 100 (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

**Note:** It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

## Mental Addition Strategies

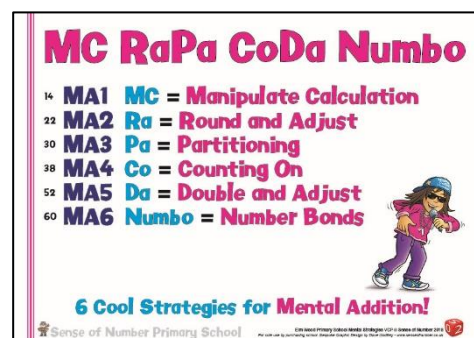
There are **6 key mental strategies** for addition, which need to be a regular and consistent part of the approach to calculation in all classes from Year 2 upwards.

These strategies will be introduced individually when appropriate, and then be rehearsed and consolidated throughout the year until they are almost second nature.

These strategies are **Partitioning, Counting On, Manipulate the Calculation, Round & Adjust, Double & Adjust** and using **Number Bonds**. The first two strategies are also part of the written calculation policy (see pages 14-18) but can equally be developed as simple mental calculation strategies once children are skilled in using them as jottings.

Using the acronym **MC RAPA CODA NUMBO**, children can be given weekly practice in choosing the most appropriate strategy whenever they are faced with a simple addition, usually of 2 or 3-digit numbers, but also spotting the opportunities when they can be used with larger numbers (E.g.  $3678 + 2997$ ) or decimals (E.g.  $4.8 + 2.2$ )

<b>MC</b>	<b>Manipulate Calculation</b>
<b>RA</b>	<b>Round &amp; Adjust</b>
<b>PA</b>	<b>Partitioning</b>
<b>CO</b>	<b>Counting On</b>
<b>DA</b>	<b>Double &amp; Adjust</b>
<b>NUMBO</b>	<b>Number Bonds</b>

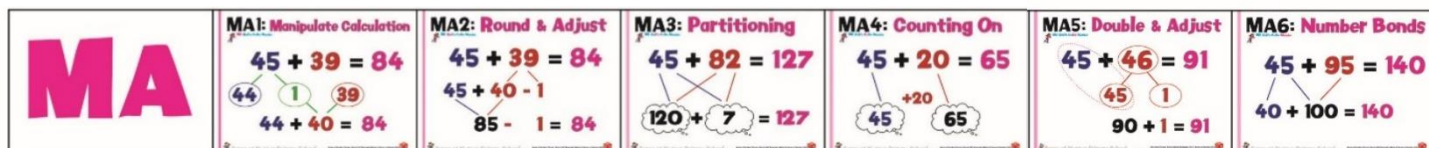


This policy not only provides examples of the 6 different strategies as mental jotting.

There is also a Concrete / Pictorial slide for each strategy, demonstrating how to give a visual picture of the strategy in question using key apparatus or manipulatives (usually Base 10). The visual slide is to give the children conceptual understanding of the mental strategy so that they can picture it before starting to write it down or use it mentally.



For example, using the number 45 (see examples below), we can look at the other number chosen, and decide on the most appropriate mental calculation strategy.

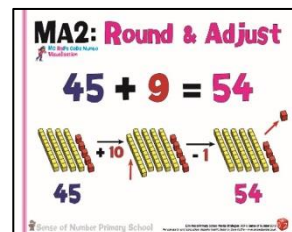
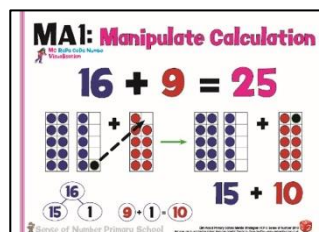


## Manipulate the Calculation / Round & Adjust.

Both of these strategies use the same mathematical thinking but in a slightly different way.

The first 2 slides show the strategies using resources.

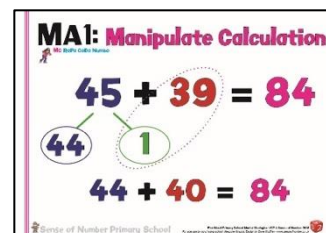
$16 + 9$  in Ten Frames shows that '1' of the counters from the '16' can be 'given' to the '9' so that there will now be  $15 + 10$ . You have 'manipulated the calculation' to make it easier.



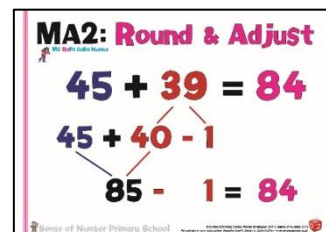
$45 + 9$  with Base 10 shows that it is much easier to add a '10' and remove a '1' rather than count on / add nine '1's'

For  $45 + 39$ , it would not be as efficient to partition or count on (though it would be relatively easy). 39 is only 1 away from 40 so there are two alternatives.

We could simply make an easier calculation (Manipulate the Calculation) by 'passing 1' from the 45 to the 39. This would give us a new calculation of  $44 + 40$ .

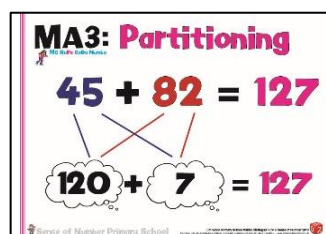


Alternatively, we could use the Round & Adjust strategy of adding 40 to the 45 then subtracting 1, as advocated by the National Numeracy Strategy.

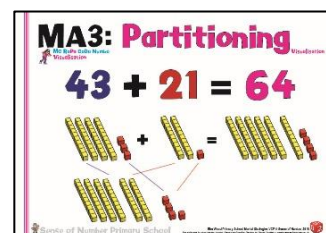


## Partitioning

The first slide shows a straightforward partition for  $43 + 21$  using Base 10. The four 10s are added to the two 10s and the three 1s are added to the single 1, making six 10s and four 1s (or 64).



For  $45 + 82$ , the strategy is a simple Partition Jot, adding the four 10s (40) to the eight 10s (80) and the five 1s (5) to the two 1s (2), making twelve 10s (120) and seven 1s (7) or 127



## Counting On

The first slide shows how easily children can demonstrate the Count On strategy using Base 10. This resource really allows the children to see what 'counting on' in 10s actually looks like. It isn't just a verbal number pattern '45, 55, 65' but it really represents four 10s (40) and five 1s (5) to which two 10s are added, giving a total of six 10s and five 1s (65).

As a mental strategy this is usually represented by a straightforward 'count on in my head' picture or, as will be seen later in the written methods section, a number line image.

**MA4: Counting On**  
 $45 + 20 = 65$   
A diagram shows a cloud containing '45' and another cloud containing '20'. An arrow labeled '+20' points from the '45' cloud to a larger cloud containing '65'.

**MA4: Counting On**  
 $45 + 20 = 65$   
A diagram shows base 10 blocks representing 45 (4 tens rods and 5 ones units). An arrow labeled '+10' points to a second set of blocks representing 55 (5 tens rods and 5 ones units). A second arrow labeled '+10' points to a final set of blocks representing 65 (6 tens rods and 5 ones units).

## Double & Adjust

This strategy requires a real sense of number, in that children have to look at both numbers and recognise that because they are 'close together' in size it is easier to use a 'near double' in order to work out the answer to the calculation.

In Year 2, for example, spotting that  $7 + 8$  is almost 'double 7' is far easier than counting on (as long as you know the answer to double 7). The first slide demonstrates this using Numicon / Number Frame pieces to visualise the 'near double', which can then be adjusted.

For  $45 + 46$ , it should then become clear that it is much easier / more efficient to use the known fact of 'Double 45' then add an extra 1 rather than partitioning ( $80 + 11$ ) or counting on ( $45 + 40 + 6$ ). If, however, doubles haven't been yet learned then one of the other strategies will need to be used instead.

**MA5: Double & Adjust**  
 $45 + 46 = 91$   
A diagram shows a cloud containing '45' and another cloud containing '46'. An arrow labeled '+1' points from the '46' cloud to a larger cloud containing '91'. Below this, the equation  $90 + 1 = 91$  is shown.

**MA5: Double & Adjust**  
 $7 + 8 = 15$   
A diagram shows Numicon pieces representing 7 (one blue piece and six red pieces) and 8 (one blue piece and seven red pieces). An arrow labeled '+1' points from the 8 pieces to a final set of pieces representing 15 (one blue piece and 14 red pieces). Below this, the equation  $7 + 8 = 7 + 7 + 1 = 14 + 1 = 15$  is shown.

## Number Bonds

In a similar way to 'Double & Adjust' the Number Bonds strategy requires the children to know and recognise key number bonds to 10 / 100 / multiples of 10, and use these to make a calculation easier. The Year 2 calculation  $3 + 4 + 7$  can be simplified by immediately making a bond to 10 then adding 4 (as shown on the slide using the Numicon pieces).

For  $45 + 95$ , it is more efficient and sensible to make a number bond of 100 using the 95 and 5 and then adding the 40 for a total of 140.

**MA6: Number Bonds**  
 $3 + 4 + 7 = 14$   
A diagram shows Numicon pieces representing 3 (one blue piece and two red pieces), 4 (one blue piece and three red pieces), and 7 (one blue piece and six red pieces). An arrow labeled '+3' points from the 4 pieces to a larger set of pieces representing 14 (one blue piece and 13 red pieces).

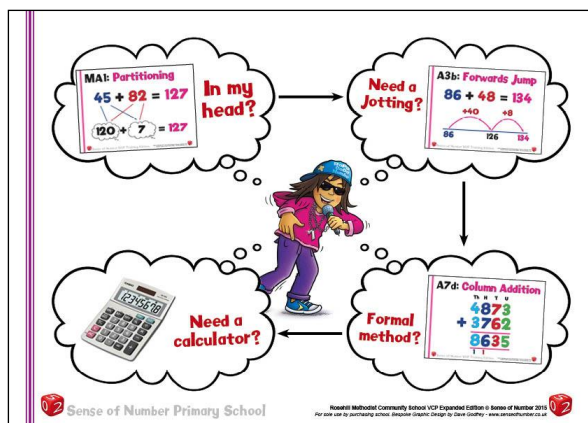
**MA6: Number Bonds**  
 $45 + 95 = 140$   
A diagram shows a cloud containing '45' and another cloud containing '95'. An arrow labeled '+5' points from the '95' cloud to a larger cloud containing '140'. Below this, the equation  $40 + 100 = 140$  is shown.



The grid below shows a complete overview of the 6 main mental calculation strategies across the year groups. It displays examples from the Visual Calculation Policy for each method from Year 1 – Year 6. In it, you can see the progression of each strategy using appropriate examples of numbers as exemplification.

	MA1: Manipulate Calculation	MA2: Round & Adjust	MA3: Partitioning	MA4: Counting On	MA5: Double & Adjust	MA6: Number Bonds
<b>Y1</b>	$16 + 9 = 25$ $15 + 10 = 25$	$45 + 9 = 54$ $45 + 10 - 1 = 54$ $55 - 1 = 54$	$43 + 21 = 64$ $8 + 2 + 4 = 14$	$45 + 20 = 65$ $12 + 5 = 17$	$7 + 8 = 15$ $5 + 6 = 11$ $10 + 1 = 11$	$3 + 4 + 7 = 14$
<b>Y2</b>	$45 + 19 = 64$ $44 + 20 = 64$	$45 + 19 = 64$ $45 + 20 - 1 = 64$ $65 - 1 = 64$	$43 + 21 = 64$ $60 + 4 = 64$	$78 + 7 = 85$ $78 + 85$	$58 + 40 = 98$ $58 + 98$	$7 + 8 = 15$ $3 + 5 = 8$ $10 + 5 = 15$ $3 + 4 + 7 = 14$ $10 + 4$
<b>Y3</b>	$45 + 97 = 142$ $42 + 100 = 142$	$45 + 97 = 142$ $45 + 100 - 3 = 142$ $145 - 3 = 142$	$57 + 25 = 82$ $70 + 12 = 82$	$85 + 50 = 135$ $85 + 135$	$534 + 300 = 834$ $534 + 834$	$16 + 17 = 33$ $16 + 1 = 17$ $32 + 1 = 33$ $43 + 9 + 7 + 21 = 80$ $50 + 30$
<b>Y4</b>	$345 + 298 = 643$ $343 + 300 = 643$	$345 + 298 = 643$ $345 + 300 - 2 = 643$ $645 - 2 = 643$	$648 + 231 = 879$ $800 + 70 + 9 = 879$	$784 + 60 = 844$ $784 + 844$	$4837 + 3000 = 7837$ $4837 + 7837$	$37 + 38 = 75$ $37 + 1 = 38$ $74 + 1 = 75$ $42 + 16 + 28 + 54 = 140$ $70 + 70$
<b>Y5</b>	$4645 + 1996 = 6641$ $4641 + 2000 = 6641$	$4645 + 1996 = 6641$ $4645 + 2000 - 4 = 6641$ $6645 - 4 = 6641$	$576 + 258 = 834$ $700 + 120 + 14 = 834$	$837 + 500 = 1337$ $837 + 1337$	$7583 + 5000 = 12583$ $7583 + 12583$	$125 + 127 = 252$ $125 + 2 = 127$ $250 + 2 = 252$ $£4.56 + £3.27 + £1.44 = £9.27$ $£6.00 + £3.27$
<b>Y6</b>	$45.2 + 49.9 = 95.1$ $45.1 + 50 = 95.1$	$45.2 + 49.9 = 95.1$ $45.2 + 50 - 0.1 = 95.1$ $95.2 - 0.1 = 95.1$	$4.73 + 2.21 = 6.94$ $6 + 0.9 + 0.04 = 6.94$	$48,926 + 30,000 = 78,926$ $48,926 + 78,926$	$5,763,947 + 4,000,000 = 9,763,947$ $5,763,947 + 9,763,947$	$4.5 + 4.7 = 9.2$ $4.5 + 0.2 = 4.7$ $9 + 0.2 = 9.2$ $24.25 + 31.63 + 21.75 = 77.63$ $46 + 31.63$

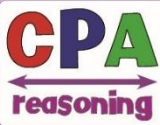
The 6 key strategies need to be linked to the key messages from pages 2 and 3 –  
 The choice as to whether a child will choose to use a mental method, or a jotting will depend upon  
 a) the numbers chosen and  
 b) the level of maths that the child is working at.  
 For example, for  $57 + 35$   
 a Year 2 child may use a long jotting or number line  
 a Year 3 child might jot down a quick partition jotting,  
 a Year 4 child could simply partition and add mentally.



As a strategy develops, a child will begin to recognise the instances when it would be appropriate:

E.g.  $27 + 9$ ,  $434 + 197$ ,  $7.6 + 1.9$  and  $5.86 + 3.97$  can all be calculated very quickly by using the **Round & Adjust** strategy.


# Models of Addition



**CPA**  
reasoning


Models of

# Addition



## Aggregation


Combining two sets of objects  
(counting all method!)




“If I have 4 blue footballs and 2 red footballs in my bag, how many footballs do I have all together?” “6”

## Augmentation

Adding to a set  
(counting on method!)




“If I had 4 footballs and then found 2 more, how many footballs would I have in total?” “6”



A.N. Other Primary School

A.N. Other Primary School Concrete & Pictorial VCP © Sense of Number 2017  
For sole use by purchasing school. Bespoke Graphic Design by Dave Godfrey - www.senseofnumber.co.uk



As the images above show, it is important that children are introduced to the two main models for addition using practical resources.

In EYFS this would be real objects such as footballs, shells, cakes, cars, dinosaurs etc.

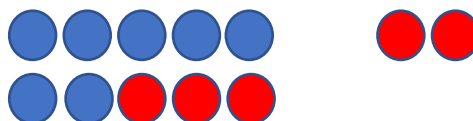
In KS1 this would progress from concrete materials such as counters or cubes to pictorial models and sketches.

The two models of addition, aggregation and augmentation will develop into the two main mental / written methods of partitioning (aggregation) and counting on (augmentation).

In simplistic terms, aggregation involves 2 separate groups that are combined to give an 'aggregate' total. E.g. There are 7 cakes in the first box and 5 cakes in the second box. How many cakes are there altogether?

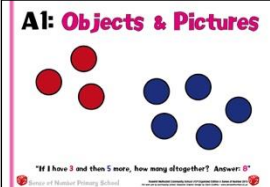
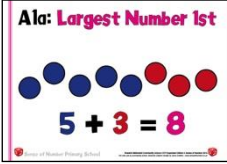
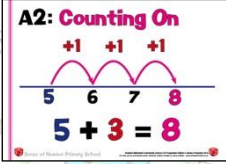
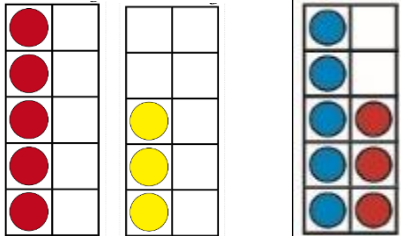


Augmentation, however, involves adding on to an existing group. E.g. There are 7 cakes in a box. My friend gives me 5 more cakes. How many cakes do I have now?

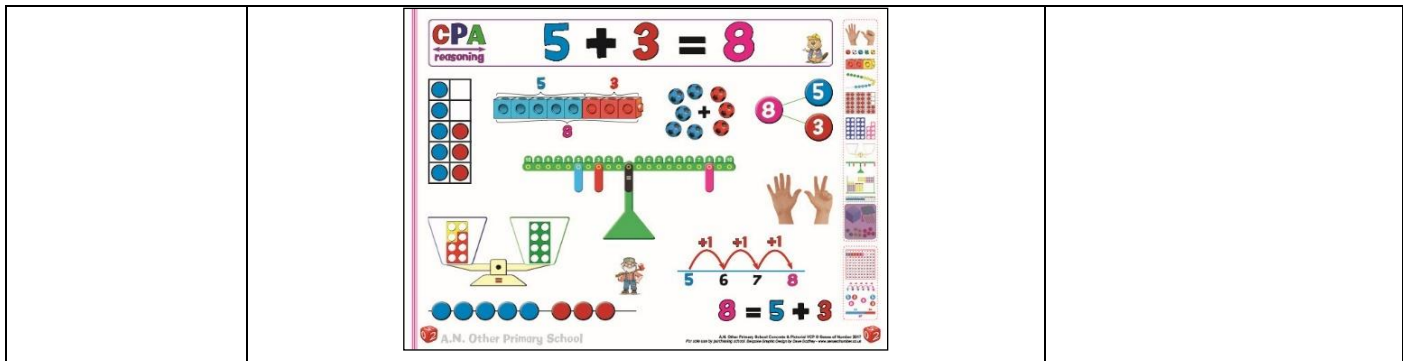


Both of these principles are explored in the actual written calculation policy below

# Written Methods of Addition

Stage 1	Finding a Total and the Empty Number Line	Alternative Method: Counting on Mentally or as a jotting
<b>FS/Y1</b>	Initially, children need to represent addition using a range of different resources and understand that a total can be found by counting out the first number, counting out the second number then counting how many there are altogether.	
	 <p><b>A1: Objects &amp; Pictures</b></p> <p>This is augmentation (see above) and can be done in either order (3+5 or 5+3)</p>	3 (held in head) then use fingers to count on 5 ("3... 4,5,6,7,8")
	<p>This will quickly develop into placing the largest number first, either as a pictorial / visual method or by using a number line. The use of the number line, however, should be delayed until the children are completely secure in their understanding of 5 + 3. Otherwise it becomes a tool that limits their understanding of what they are actually representing</p>  	5 (held in head) then count on 3 ("5 ... 6, 7, 8")
	<p>The 'Egg Box' / 'Ten Frame' image is an excellent visual tool to support both models of addition</p> <p style="text-align: center;">Aggregation      Augmentation</p> 	
	<p>Before moving onto jottings or written methods (for any calculation), children need to be shown a wide range of images that support their understanding.</p> <p>For simple calculations, the 'equals' sign can be viewed as a way to show the total (egg boxes, cubes, footballs, fingers) but also as a 'balance' (Numicon, peg balance), where 5 and 3 have a value that is equal to / the same as, or that balances 8.</p>	

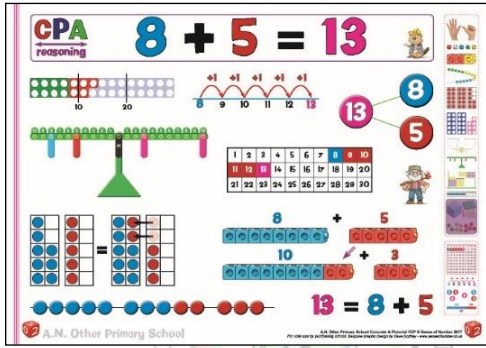




**Y1/2**

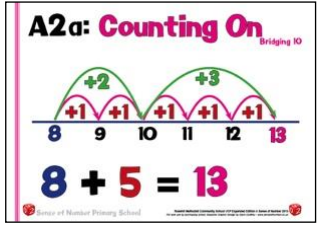
When bridging through 10, a calculation can be seen as a straightforward 'count on' process (**see number line and 100 square images below**), a balance image as before (see balance and number bond diagram) but, more importantly as a strategy where the '5' is partitioned into 2 and 3 (see Numicon, multi-link and Ten Frame images).

In effect, for these images / strategies, the calculation is being re-written as  $8 + 5 = (8 + 2) + 3 = 10 + 3 = 13$



When developing the concrete picture into a jotting the number line can be used to display the 'count on'

$(8 + 1 + 1 + 1 + 1 + 1)$   
or the partition  $(8 + 2 + 3)$ .



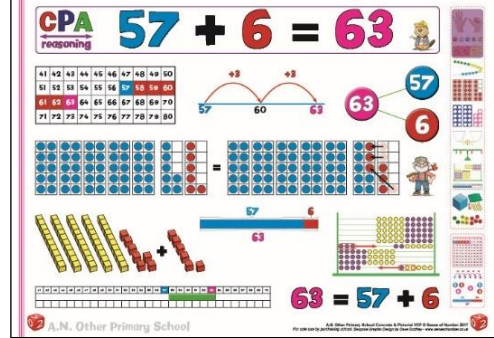
Once this image is secure, the same strategies can be done mentally: -

8 (in head) then count on 5  
 ("8 ... 9, 10, 11, 12, 13")  
 Or " $8 + 2 = 10...$   
 $10 + 3 = 13$ ).

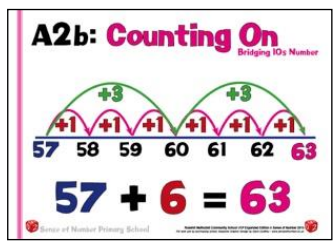
The next step is to bridge through a multiple of 10. As above, the models and images show different strategies for finding the total or balancing the equation.

The 100 square & number rod images show counting on in 1s  
 Tens Frames show a balanced equation created by passing over some counters from the 6 to the 57.

Base 10 is used to show partitioning. The 1s have been set out Tens Frame style to visualise how the 5s can be combined to make a 10.



Again, the number line jotting can display counting on  $(57+1+1+1+1+1+1)$  or partitioning  $(57 + 3 + 3)$



This picture then becomes the mental strategies: -

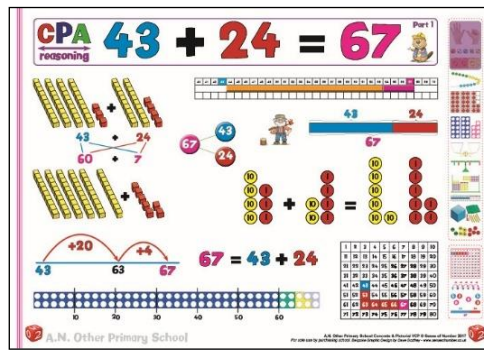
57 (in head) - count on 6  
 ("**57, 58, 59, 60, 61, 62, 63**")  
 Or " $57 + 3 = 60 ...$   
 $60 + 3 = 63$ )

There are a wide range of ways in which children can explore / visualize / create two-digit calculations.  
 The Base 10 and Place Value Counters demonstrate partitioning and recombining (adding the 10s and adding the 1s)  
 Numicon shows the 'answer' once the 10s & 1s are combined.  
 Th number rods and 100 square display counting on 20 then 4.

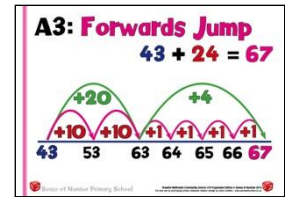
The number line shows how to keep one number whole, whilst partitioning the other number.



There is also a part/whole bar model and number bond diagram.



Firstly, add the tens then the ones individually

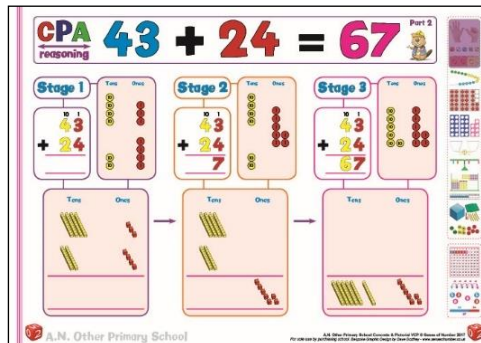


$(43 + 24 =$   
 $43+10+10+1+1+1+1)$   
 before counting on in tens  
 and ones ( $43 + 20 = 63 ..$   
 $63 + 4 = 67)$

The most crucial element of teaching addition is to ensure that the children have a 'picture' of partitioning and exchanging / regrouping, meaning that they can visualize the calculation before they move on to written / column procedures.

This needs to be done practically many times before they are asked to show the 'method' in a column.

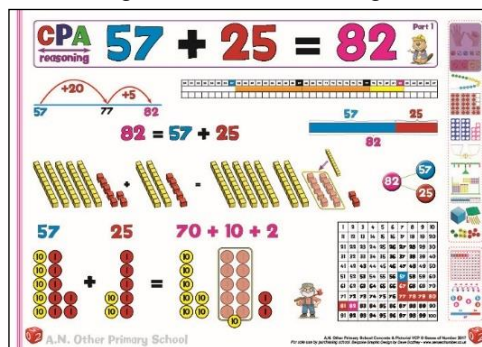
See 'column method' section for further examples of column addition using concrete materials to embed understanding.



Develop to crossing the 10s, then the 100s boundary

$57 + 25 = 82$        $86 + 48 = 134$

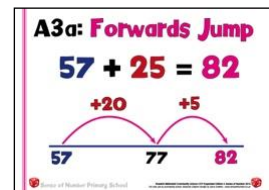
Even as the calculations increase, and start to deal with exchanging, it is important that they are initially visualised using a range of resources / images.



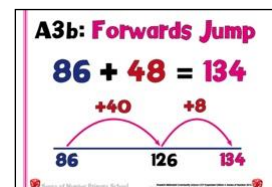
Using either Base 10 or place value counters, calculations which involve exchanging can be demonstrated.

Note the layout of the resources in Tens Frame style so that it is much easier to see the five 1s from the 57 added to the five 1s from the 25 to make an extra 10.

By this stage the number line no longer counts in individual 10s & 1s.



$57 + 20 = 77...$   
 $77 + 5 = 82$



$86 + 40 = 126...$   
 $126 + 8 = 134$

**Y3/4**

For some children, the number line method can still be used for 3-digit calculations.

$$687 + 200 = 887...$$

$$887 + 40 = 927...$$

$$927 + 8 = 935$$

Or  $687 + 200 + 40 + 8 = 935$

**A3c: Forwards Jump**

$$687 + 248 = 935$$

**Y5/6**

In Years 5 and 6, if necessary, children can return to the number line method to support their understanding of decimal calculation

$$4.8 + 3 = 7.8$$

$$7.8 + 0.8 = 8.6$$

Or

$$4.8 + 3 + 0.8 = 8.6$$

**A3f: Decimal Jump**

$$4.8 + 3.8 = 8.6$$

**A3g: Decimal Jump**

$$5.65 + 3.29 = 8.94$$

Number lines support children's thinking if they find partitioning / column addition difficult, as it simply involves counting on in 100s, 10s & 1s.

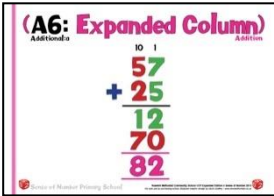
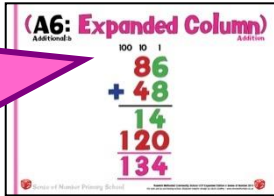
Hopefully, with the above calculation, many children would mentally Round & Adjust ( $4.8 + 4 - 0.2 = 8.6$ )

Stage 2	Partition Jot	Alternative Method: Traditional Partitioning
<b>Y2/3</b>	<p>Traditionally, partitioning has been presented using the method on the right. Although this does support place value and the use of arrow cards, it is very laborious, so it is suggested that adopting the 'partition jot' method will improve speed and consistency for mental to written (or written to mental) progression</p> <p>As soon as possible, refine this method to a much quicker and clearer 'Partition Jot' approach</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p><b>A5: Partition Jot</b></p> <math display="block">43 + 24 = 67</math> </div>	<p>Record steps in addition using partition, initially as a jotting: -</p> $43 + 24 = 40 + 20 + 3 + 4 =$ $60 + 7 = 67$ <p>Or, preferably</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p><b>A4: Partitioning</b></p> <math display="block">43 + 24 = 67</math> <math display="block">40 + 20 = 60</math> <math display="block">3 + 4 = 7</math> <hr style="width: 100%;"/> <math display="block">67</math> </div>
	<p>As before, develop these methods, especially Partition Jot, towards crossing the 10s and then 100s.</p> <div style="display: flex; justify-content: space-around; margin: 10px auto;"> <div style="border: 1px solid black; padding: 5px;"> <p><b>A5a: Partition Jot</b></p> <math display="block">57 + 25 = 82</math> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>A5b: Partition Jot</b></p> <math display="block">86 + 48 = 134</math> </div> </div>	<div style="display: flex; justify-content: space-around; margin: 10px auto;"> <div style="border: 1px solid black; padding: 5px;"> <p><b>A4b: Partitioning</b></p> <math display="block">86 + 48 = 134</math> <math display="block">80 + 40 = 120</math> <math display="block">6 + 8 = 14</math> <hr style="width: 100%;"/> <math display="block">134</math> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>A4a: Partitioning</b></p> <math display="block">57 + 25 = 82</math> <math display="block">50 + 20 = 70</math> <math display="block">7 + 5 = 12</math> <hr style="width: 100%;"/> <math display="block">82</math> </div> </div>
	<p>This method will soon become the recognised jotting to support the teaching of partitioning. It can be easily extended to 3 and even 4-digit numbers when appropriate.</p>	<p>For certain children, the traditional partitioning method can still be used for 3 digit numbers, but it is probably too laborious for 4 digit numbers.</p>
<b>Y3/4</b>	<div style="display: flex; justify-content: space-around; margin: 10px auto;"> <div style="border: 1px solid black; padding: 5px;"> <p><b>A5c: Partition Jot</b></p> <math display="block">687 + 248 = 935</math> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>A5d: Partition Jot</b></p> <math display="block">4873 + 3762 = 8635</math> </div> </div>	<div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p><b>A4c: Partitioning</b></p> <math display="block">687 + 248 = 935</math> <math display="block">600 + 200 = 800</math> <math display="block">80 + 40 = 120</math> <math display="block">7 + 8 = 15</math> <hr style="width: 100%;"/> <math display="block">935</math> </div>
	<p>Partition jot is also extremely effective as a quicker alternative to column addition for decimals.</p>	<p>Some simple decimal calculations can also be completed this way.</p>





<p><b>Y5/6</b></p>	<p><b>A5f: Partition Jot</b></p> $4.8 + 3.8 = 8.6$ $7 + 1.6$	<p><b>A5g: Partition Jot</b></p> $5.65 + 3.29 = 8.94$ $8 + 0.8 + 0.14$	
	<p>For children with higher-level decimal place value skills, partition jot can be used with more complex decimal calculations or money.</p>		
	<p><b>A5h: Partition Jot</b></p> $76.7 + 58.5 = 135.2$ $120 + 14 + 1.2$	<p><b>A5i: Partition Jot</b></p> $£38.25 + £27.46 = £65.71$ $£65.00 + £0.71$	

<p><b>Stage 3</b></p>	<p><b>Expanded Method in Columns</b></p>	
<p><b>Y3</b></p>	<p>Column methods of addition are introduced in Year 3, but it is crucial that they still see mental calculation as their first principle, especially for 2-digit numbers. Column methods should only be used for more difficult calculations, usually with 3-digit numbers that cross the Thousands boundary or most calculations involving 4-digit numbers and above.</p> <p>N.B. Even when dealing with bigger numbers / decimals, children should still look for the opportunity to calculate mentally (E.g. 4675 + 1998)</p>	
	<p>2 digit examples are used below simply to introduce column methods to the children. Most children would continue to answer these calculations mentally or using a simple jotting.</p>	
	<p>Using the column, children need to learn the principle of adding the ones first rather than the tens.</p>	
	<p>The 'expanded' method is a very effective introduction to column addition. It continues to use the partitioning strategy that the children are already familiar with, but begins to set out calculations vertically. It is particularly helpful for automatically 'dealing' with the 'carry' digit. <b>It is crucial, however, to ensure that practical apparatus has been used first before any sort of columnar procedure is introduced (see examples below in the 'Column Method' section</b></p>	
<p><b>Y3/4</b></p>	<p><b>A. Single 'carry' in units</b></p>	<p><b>B. 'Carry' in units and tens</b></p>
		 <p>'Eighty plus forty equals one hundred and twenty, because 'eight plus four equals twelve.</p>
	<p>Once this method is understood, it can quickly be adapted to using with three-digit numbers. It is rarely used for 4 digits and beyond as it becomes too unwieldy.</p>	

**A6: Expanded Column**

$$\begin{array}{r}
 100 \ 10 \ 1 \\
 687 \\
 + 248 \\
 \hline
 15 \\
 120 \\
 800 \\
 935
 \end{array}$$

The time spent on practicing the expanded method will depend on security of number facts recall and understanding of place value.

Once the children have had enough experience in using expanded addition and have also used practical resources (Base 10 / place value counters) to model exchanging in columns, they can be taken on to standard, 'traditional' column addition.

## Stage 4

## Column Method

**Y3/4**

As with the expanded method, begin with 2-digit numbers, simply to demonstrate the method, before moving to 3-digit numbers.

**Make it very clear to the children that they are still expected to deal with all 2-digit (and many 3 digit) calculations mentally (or with a jotting), and that the column method is designed for numbers that are too difficult to access using these ways. The column procedure is not intended for use with 2-digit numbers unless completely necessary for certain children who have no other strategy.**

**'Carry' ones then ones and tens.**

The most important part of the transition from mental and jotting approaches towards the use of column methods is the initial use of concrete materials to make / create and discuss the understanding behind the method. If children have had regular experience of using **Base 10 apparatus** and then **Place Value Counters** to make and solve calculations, (especially the regrouping of 10 'ones' into 1 'Ten' and 10 'Tens' into 1 'Hundred') then the column method is simply a written version of what they already understand.

The image shows three CPA Reasoning worksheets. The first shows 57 + 25 = 82 with three stages of using base 10 blocks. The second shows 86 + 48 = 134 with four stages. The third shows 687 + 248 = 935 with four stages. Each stage shows how concrete materials are used to model the addition and how the result is written in a columnar format.

**(A7: Column Addition)**

$$\begin{array}{r}
 10 \ 1 \\
 57 \\
 + 25 \\
 \hline
 82 \\
 1
 \end{array}$$

**(A7: Column Addition)**

$$\begin{array}{r}
 100 \ 10 \ 1 \\
 86 \\
 + 48 \\
 \hline
 134 \\
 1 \ 1
 \end{array}$$

**A7: Column Addition**

$$\begin{array}{r}
 100 \ 10 \ 1 \\
 687 \\
 + 248 \\
 \hline
 935 \\
 1 \ 1
 \end{array}$$

Record carry digits below the line.

Use the words 'carry ten' and 'carry hundred', not 'carry one'

**Y4**

Once confident, use the column method with 4-digit numbers. At this point the apparatus is not necessary as calculation procedure should be fully understood.

**A7d: Column Addition**

$$\begin{array}{r} 4873 \\ + 3762 \\ \hline 8635 \end{array}$$

**Y5/6**

Extend column methods to 5/6-digit calculations then decimal calculations (**Year 5**). Even with decimals, it makes sense to embed the CPA approach, using Decimal Place Value Counters to create and practice the calculation first, explaining the method and describing the principles which takes place when the children are exchanging / regrouping tenths and hundredths

**CPA reasoning**  $4.8 + 3.8 = 8.6$

Elm Wood Primary School

**CPA reasoning**  $76.7 + 58.5 = 135.2$

Elm Wood Primary School

*If children make repeated errors at any stage, they can return to the expanded method or an earlier jotting.*

**A7e: Column Addition**

$$\begin{array}{r} 787567 \\ + 446278 \\ \hline 1233845 \end{array}$$

**A7f: Column Addition**

$$\begin{array}{r} 4.8 \\ + 3.8 \\ \hline 8.6 \end{array}$$

**A7g: Column Addition**

$$\begin{array}{r} 5.65 \\ + 3.29 \\ \hline 8.94 \end{array}$$

**A7h: Column Addition**

$$\begin{array}{r} 10 \quad 1 \quad \frac{1}{10} \\ 76.7 \\ + 58.5 \\ \hline 135.2 \end{array}$$

**A7i: Column Addition**

$$\begin{array}{r} \text{With Money} \\ \text{£}38.25 \\ + \text{£}27.46 \\ \hline \text{£}65.71 \end{array}$$

The key skill in upper Key Stage 2 that needs to then be developed, usually in Year 6, is the laying out of the column method for calculations with decimals in different places.

**A7j: Column Addition**

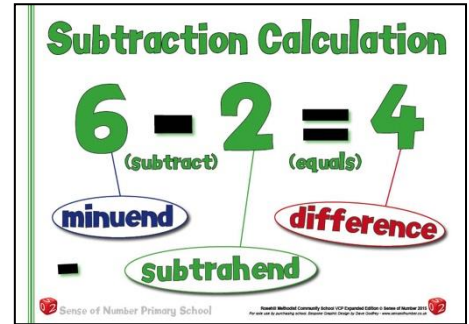
$$\begin{array}{r} 10 \quad 1 \quad \frac{1}{10} \quad \frac{1}{100} \\ 73.4 \\ + 5.67 \\ \hline 79.07 \end{array}$$





# Subtraction Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.



To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as  $160 - 70$ ) using the related subtraction fact (e.g.  $16 - 7$ ), and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into  $70 + 4$  or  $60 + 14$ ).

*Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.*

## Mental Subtraction Strategies

There are **6 key mental strategies** for subtraction, which need to be a regular and consistent part of the approach to calculation in all classes from Year 2 upwards.

These strategies will be introduced individually when appropriate, and then be rehearsed and consolidated throughout the year until they are almost second nature.

These strategies are **Counting On, Counting Back, Partitioning, Manipulate the Calculation, Round & Adjust** and using **Number Facts**. The first two strategies are also part of the written calculation policy (see pages 14-18) but can equally be developed as simple mental calculation strategies once children are skilled in using them as jottings.

Using the acronym **MC RAPa CoOCoB NumFa**, children can be given weekly practice in choosing the most appropriate strategy whenever they are faced with a simple subtraction, usually of 2 or 3 digit numbers, but also spotting the opportunities when they can be used with larger numbers (E.g.  $3678 + 2997$ ) or decimals (E.g.  $4.8 + 2.2$ )

**MC RaPa CoOCoB NumFa**

69 **MS1 MC = Manipulate Calculation**

77 **MS2 Ra = Round and Adjust**

85 **MS3 Pa = Partitioning**

91 **MS4 CoO = Counting On**

108 **MS5 CoB = Counting Back**

123 **MS6 NumFa = Number Facts**

**6 Cool Strategies for Mental Subtraction!**

Sense of Number Primary School

Use of Number Primary School Mental Strategies VCP © Sense of Number 2016  
For more see the calculation policy document located in the link below: www.senseofnumber.co.uk



This policy not only provides examples of the 6 different strategies as mental jottings. There is also a Concrete / Pictorial slide for each strategy, demonstrating how to give a visual picture of the strategy in question using key apparatus or manipulatives (usually Base 10). The visual slide is to give the children conceptual understanding of the mental strategy so that they can picture it before starting to write it down or use it mentally.

Children need to acquire **one efficient written method of calculation for subtraction**, which they know they can rely on **when mental methods are not appropriate**.

**NOTE:** They should look at the actual numbers each time they see a calculation and decide whether or not their favoured method is most appropriate (e.g. If there are zeroes in a calculation such as  $206 - 198$ ) then the 'counting on' approach may well be the best method in that particular instance).

Therefore, when subtracting, whether mental or written, children will mainly choose between two main strategies to find the difference between two numbers: -

### **Counting Back (Taking away)**

When should we count back and when should we count on?

This will alter depending on the calculation (see below), but often the following rules apply;

### **Counting On**

*If the numbers are far apart, or there isn't much to subtract ( $278 - 24$ ) then count back.*



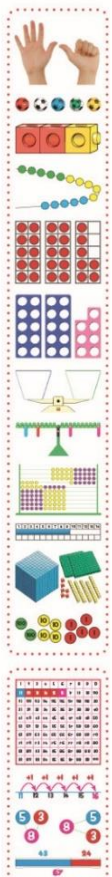
*If the numbers are close together ( $206 - 188$ ), then count up*

*In many cases, either strategy would be suitable, depending on preference ( $743 - 476$ )*

# Models of Subtraction


**CPA** Models of **Subtraction**

reasoning of



## Removing Items


**Take Away**



"If I had 8 footballs and kicked 3 over the fence, how many did I have left?" "5"

**Reduction**


$8 - 3 = 5$



"The football cost £8 but I got £3 off in the sales. How much did I pay?" "£5"

## Comparing Sets

**Comparison**




$9 - 6 = 3$

"If I had 9 footballs and you had 6, how many more balls have I got than you?" "3"


**Inverse of Addition**

"The football costs £9 but I've only got £6. How much more do I need?" "£3"




$£6 + ? = £9$

## Whole / Part / Part




"There are 10 footballs in my bag. 6 are red, how many are yellow?" "4"

$10 - 6 = 4$



A.N. Other Primary School

A.N. Other Primary School Concrete & Pictorial VCP © Sense of Number 2017  
For sole use by purchasing school. Bespoke Graphic Design by Dave Godfrey - www.senseofnumber.co.uk



It is fairly common for children (and teachers) to view subtraction in one simplistic way – as a ‘take away’. Even when reading subtraction calculations such as  $42 - 38$  the most common way in which it is read aloud would be ‘42 take away 38’.

Although we could count back 30 then count back 8 from 42 (with or without a number line), and might even count out 42 items then remove 38 of them (Base 10 or otherwise), the easiest / most efficient way to solve the calculation would be to ‘count on’ 4 from 38 to 42 (or simply to recognise the difference of 4).

Therefore, it is vital that everybody, both teachers and children, are encouraged / persuaded to read all calculations using the correct language (‘42 subtract 38’ or ‘What is the difference between 42 and 38?’) and then allow the person answering the question the opportunity to choose the most appropriate strategy.

As can be seen in the later parts of this section, the ‘count on’ approach is usually demonstrated via the use of a **number line or jotting**, whilst the ‘take away’ approach is explored using **Base 10 apparatus** and **Place Value Counters** (as with addition) to allow visualisation and understanding of the exchanging / regrouping principles.





# Written Methods of Subtraction

INTRO	Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
<p><b>FS/Y1</b></p>	<p>Early subtraction in EYFS will primarily be concerned with '<b>taking away</b>', and will be modelled using a wide range of models and resources. These will usually be natural resources and real-life objects, and will often be part of a story telling scenario where the children can 'make' subtraction to tell the story</p>	
	<p>This will continue in Year 1, using resources such as multi-link cubes, Numicon, Ten Frames, bead strings and pictures (like the footballs shown below) to model the 'take away' approach to subtraction.</p> <p>Once this is secure then images such as the desktop number track / line can be used to practice taking away practically, and then developed into counting back on demarcated number lines.</p> <div data-bbox="437 1043 820 1317" data-label="Complex-Block"> <p><b>S1: Objects</b></p> <p><math>7 - 3 = 4</math></p> <p>"What do I get if I take 3 away from 7? Answer: 4"</p> <p><small>Sense of Number Primary School</small></p> </div> <div data-bbox="387 1323 874 1664" data-label="Complex-Block"> <p><b>CPA reasoning</b> <math>7 - 3 = 4</math></p> <p><b>Count Back</b> ← Images</p> <p><math>4 = 7 - 3</math></p> <p><small>A.N. Other Primary School</small></p> </div>	<p>In Year 1, it is also vital that children understand the concept of subtraction as 'finding a difference' by <b>comparison</b> and realise that <b>any</b> subtraction can be answered in 2 different ways, either by counting up or counting back.</p> <p>Again, this needs to be modelled and consolidated regularly using a wide range of resources, especially multilink towers, counters and Numicon. The images below also show the early version of bar modelling, as well as the Numicon pieces being used to demonstrate the equals sign as a balance (<math>7</math> is equal to <math>5 + \underline{\quad}</math>)</p> <div data-bbox="1018 1048 1401 1317" data-label="Complex-Block"> <p><b>S2: What's the Difference?</b></p> <p><math>7 - 5 = 2</math></p> <p>"How many more is 7 than 5? What is the difference?"</p> <p><small>Sense of Number Primary School</small></p> </div> <div data-bbox="954 1323 1441 1664" data-label="Complex-Block"> <p><b>CPA reasoning</b> <math>7 - 5 = 2</math></p> <p><b>Count On</b> Images →</p> <p><math>2 = 7 - 5</math></p> <p><small>A.N. Other Primary School</small></p> </div>

# Stage 1

# Using the empty number line

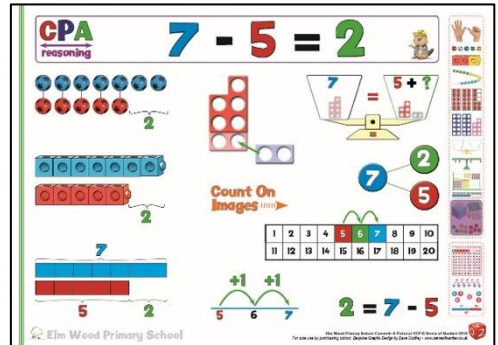
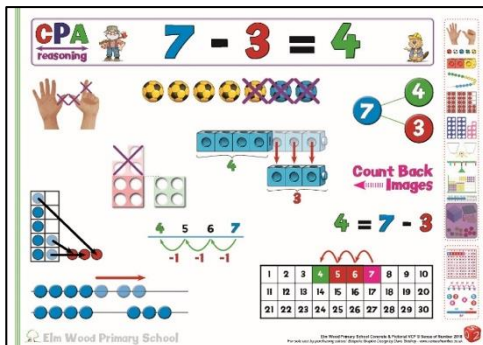
## Subtraction by counting back (or taking away)

## Subtraction by counting up (or complementary addition)

Before developing any jottings, mental methods or column procedures, children need to explore, discuss and visualize the two main models of subtraction using a wide range of models, images and apparatus. (including Numicon, multi-link cubes and Tens Frames). It is not advised to begin using a number line as a tool for calculation until the children understand the principles behind why it is being used. Otherwise the number line becomes a method, rather than a tool to embed and further understanding and fluency.

Images for 'counting back'

Images for 'counting on'

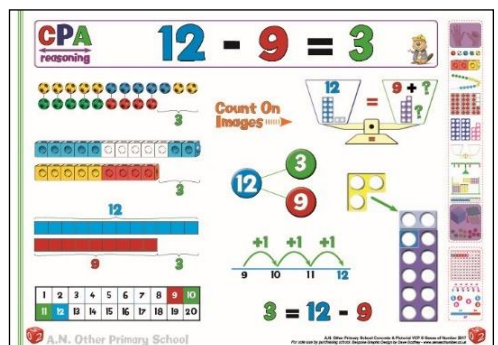
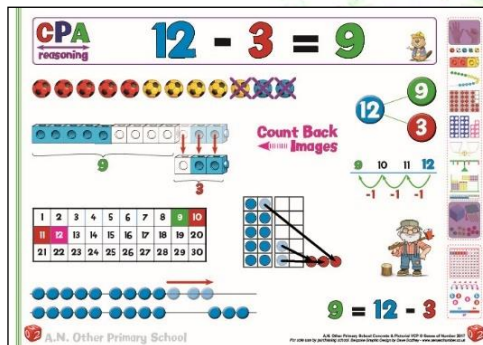


# Y1

Before using a number line, therefore, children need to be given the opportunity to 'make' both models of subtraction using resources.

For  $12 - 3$ , the multi-link and Ten Frame pictures clearly show  $12 - 2 - 1$ , whilst the footballs display the 'take away'.

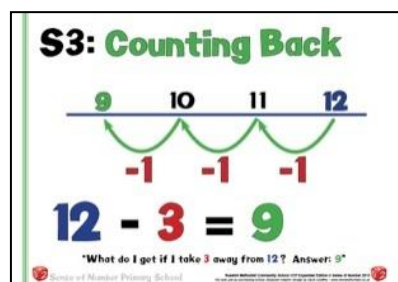
For  $12 - 9$  the number rods, cubes and footballs display a clear comparison, where the difference of 3 can be seen.



The empty number line helps to record or explain the steps in mental subtraction. It is an ideal model for **counting back** and **bridging ten**, as the steps can be shown clearly. It can also show **counting up** from the smaller to the larger number to **find the difference**.

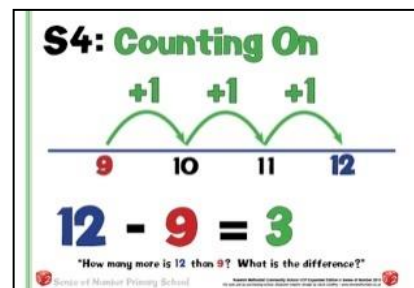
The steps often bridge through a multiple of 10.

$$12 - 3 = 9$$



Small differences can be found by counting up

$$12 - 9 = 3$$

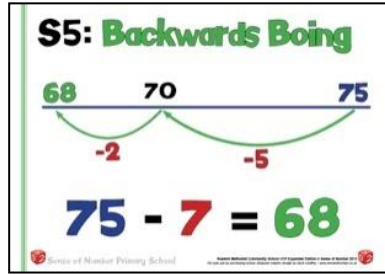




# Y2/3

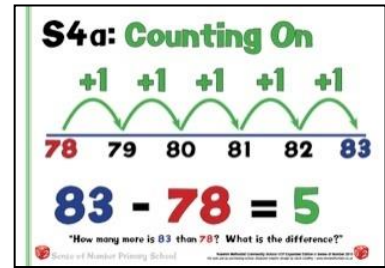
This is developed into crossing any multiple of 10 boundary.

$$75 - 7 = 68$$

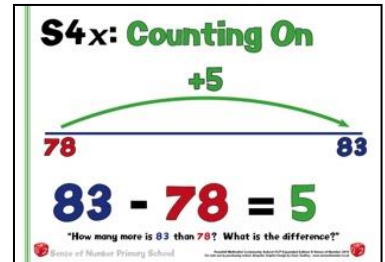


For 2 (or 3) digit numbers close together, count up. First, count in ones

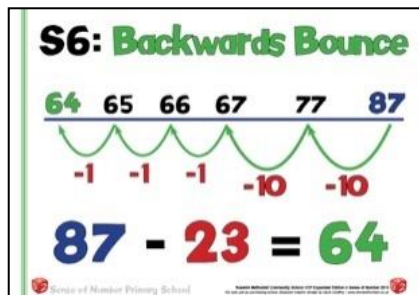
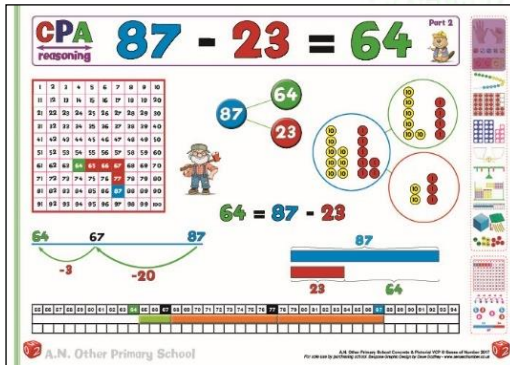
$$83 - 78 = 5$$



Then, use number facts to count up in a single jump



For 2-digit numbers, count back in 10s and 1s. As with addition, it is important to let the children manipulate materials in order to 'see' the calculation. Base 10 and Place Value Counters are particularly crucial as these are images which are used throughout the topic (see expanded method section for these images). The image below uses Place Value Counters to demonstrate the Number Bond diagram. Once secure, model the use of the empty number line where tens and ones are subtracted in single jumps ( $87 - 20 - 3$ )

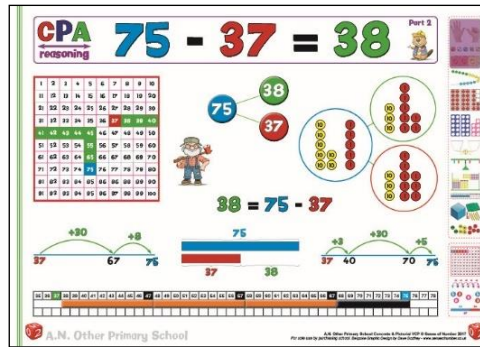


Continue to spot small differences with 3-digit numbers

$$(403 - 397 = 6)$$

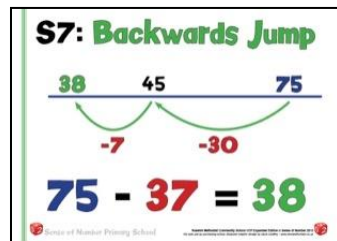

**Some numbers (75 – 37) can be subtracted just as quickly either way.**

The images below show 75-37 as a Number Bond Diagram (also visualised with Place Value Counters), as a 'count up' image using the number line (explored further below in 2 different ways) and number rods, and as a take away / count back image on the 100 Square.

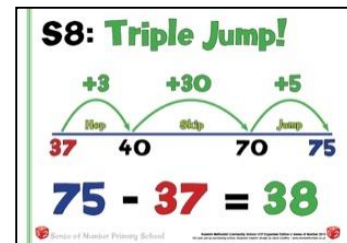


The number line itself is an excellent visual jotting for both counting on and counting back, depending on the children's preferred strategy: -

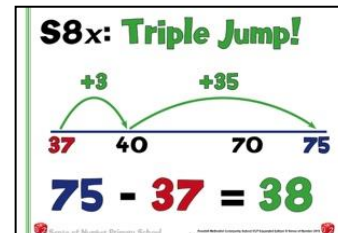
Either count back 30 then count back 7



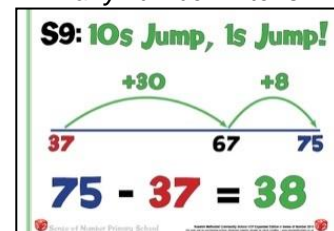
Or count up from smaller to the larger number, initially with a 'triple jump' strategy of jumping to the next 10, then multiples of 10, then to the target number.



This can also be done in 2 jumps.



Some children prefer to jump in tens and ones, which is an equally valid strategy, as it links to the mental skill of 'counting up from any number in tens'



# Stage 2

# Expanded Method & Number Lines (continued)

## Subtraction by counting back Expanded Method

## Subtraction by counting up Number Lines (continued)

In Year 3, according to the New Curriculum, children are expected to be able to use both jottings and written column methods to deal with 3-digit subtractions.

This is only guidance, however – as long as children leave Year 6 able to access all four operations using formal methods, schools can make their own decisions as to when these are introduced.

It is very important that they have had regular opportunities to use the number line 'counting up' approach first (right hand column below) so that they already have a secure method that is almost their first principle for most 2 and 3-digit subtractions.

This means that once they have been introduced to the column method they have an alternative approach that is often preferable, depending upon the numbers involved.

The number line method also gives those children who can't remember or successfully apply the column method an approach that will work with any numbers (even 4-digit numbers and decimals) if needed.

It is advisable to spend at least the first term in Year 3 focusing upon the number line / counting up approach as a jotting through regular practice, while resources such as Base 10 are being used to explore decomposition practically. The column method can then be introduced in the 2<sup>nd</sup> / 3<sup>rd</sup> term once the understanding is secure.

Ideally, whenever columns are introduced, the expanded method should be practiced in depth (potentially up until 4-digit calculations are introduced). This should be done firstly with apparatus to build up a visual picture, and then gradually developed into the column procedure.

# Y3/4

The expanded method of subtraction is an excellent way to introduce the column approach as it maintains the place value and is much easier to model practically with place value equipment such as Base 10 or place value counters.

Introduce the expanded method with 2-digit numbers, but only to explain the process.

Column methods are very rarely needed for 2-digit calculations.

Give the children ample opportunity to extend their place value skills into column subtraction by 'making' the calculation and explaining the process before writing it down.

Partition both numbers into tens & ones, firstly with no exchange then exchanging from tens to ones.

CPA reasoning **87 - 23 = 64** Part 1

1.  $87 = 80 + 7$ ,  $23 = 20 + 3$

2.  $80 - 20 = 60$

3.  $7 - 3 = 4$

4.  $60 + 4 = 64$

5.  $64 = 87 - 23$

Elm Wood Primary School





Make sure that children are explaining the process of exchanging / regrouping whilst using the materials. Give them many opportunities to take the 1 Ten and exchange / regroup as 10 Ones before writing this down in expanded form.

$$87 - 23$$

$$75 - 37$$

**(S10: Expanded Column)**  
Addition  
Subtraction

$$87 - 23 = 64$$

80	7
20	3
60	4

**(S10: Expanded Column)**  
Addition  
Subtraction

$$75 - 37 = 38$$

<del>70</del> <sup>60</sup>	15
30	7
30	8

Develop into exchanging from hundreds to tens and tens to ones, always 'making' and discussing with apparatus first. Be explicit in explaining how the apparatus leads to the column procedure.

N.B. Please note the layout of the Base 10 resources in Tens Frame style, so that the groups of 5 can be seen immediately and don't need to be re-counted.

$$132 - 56$$

**(S10: Expanded Column)**  
Addition  
Subtraction

$$132 - 56 = 76$$

<del>100</del> <sup>0</sup>	<del>30</del> <sup>120</sup>	1
-	50	6
	70	6

The number line method is equally as effective when crossing the hundreds boundary, either by the triple / quad jump strategy or by counting in tens then ones.

**S8b: Quad Jump!**

$$132 - 56 = 76$$

**S9b: 10s Jump, 1s Jump!**

$$132 - 56 = 76$$

The 'quad jump' can be completed by many children in fewer steps, either a triple or **double** jump.

**S8x: Quad Jump!**

$$132 - 56 = 76$$

*For examples without exchanging, the number line method takes considerably longer than mental partitioning or expanded.*



**A**

Develop the method into three-digit numbers.  
Subtract the ones, then the tens, then the hundreds.

**S10x: Expanded Column**  
Subtraction (100, 10, 1s)

$$784 - 351 = 433$$

700	80	4
- 300	50	1
400	30	3

Demonstrate without exchanging first.

**B**

Move towards exchanging from hundreds to tens and tens to ones, in two stages if necessary. Use practical apparatus first at all times, especially when dealing with 3-digit calculations.

**CPA Reasoning**  $723 - 356 = 367$  Part 1

**CPA Reasoning**  $723 - 356 = 367$  Part 2

**S10: Expanded Column**  
Subtraction (100, 10, 1s)

$$723 - 356 = 367$$

<del>700</del>	<del>20</del>	3
- 300	50	6
300	60	7

The example below shows 2 alternatives, for children who need different levels of support from the image.

**S8c: Big Jump!**

$$723 - 356 = 367$$

As before, many children prefer to count in hundreds, then tens, then ones.

**S9c: 100s, 10s, 1s Jump**

$$723 - 356 = 367$$

**C**

Use some examples which include the use of zeros

**S10: Expanded Column**  
Subtraction (100, 10, 1s)

$$605 - 328 = 277$$

500	90	1
<del>600</del>	<del>20</del>	3
- 300	20	6
300	70	7

→

*Continue to use expanded subtraction until both number facts and place value are considered to be very secure!*

*For numbers containing zeros, counting up is often the most reliable method.*

**S4x: Counting On**

$$605 - 328 = 277$$

36 – Boarshaw Primary School Sense of Number Education Calculation Policy © www.senseofnumber.co.uk

Stage 3		Standard Column Method (decomposition)																									
		Subtraction by counting back Standard Method	Subtraction by counting up Number Lines (continued)																								
<p>Mainly</p> <p><b>Y4+</b></p>	<p><b>Decomposition relies on secure understanding of the expanded method, and simply displays the same numbers in a contracted form.</b></p> <p>As with expanded method, and using practical resources such as place value counters to support the teaching, children in Years 3 or 4 (depending when the school introduces the column procedure) will quickly move from decomposition via 2-digit number 'starter' examples to 2 / 3 digit and then 3-digit columns.</p> <p style="text-align: center;"><b>75 – 37                      132 – 56</b></p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px;"> <p><b>(S11: Column Subtraction)</b></p> <math display="block">\begin{array}{r} 10 \quad 1 \\ 675 \\ - 37 \\ \hline 38 \end{array}</math> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>(S11: Column Subtraction)</b></p> <math display="block">\begin{array}{r} 100 \quad 10 \quad 1 \\ 0 \quad 12 \quad 1 \\ 132 \\ - 56 \\ \hline 76 \end{array}</math> </div> </div>																										
	<p style="text-align: center;"><b>723 – 356</b></p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p><b>S11: Column Subtraction</b></p> <math display="block">\begin{array}{r} 100 \quad 10 \quad 1 \\ 6 \quad 11 \quad 1 \\ 723 \\ - 356 \\ \hline 367 \end{array}</math> </div>	<div style="border: 2px solid purple; border-radius: 15px; padding: 10px; background-color: #e0ffe0;"> <p><i>Continue to refer to digits by their actual value, not their digit value, when explaining a calculation. E.g. One hundred and twenty subtract fifty.</i></p> </div>																									
	<p>Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2)</p> <p style="text-align: center;"><b>605 – 328</b></p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p><b>S11x: Column Subtraction</b></p> <table style="margin: auto; border-collapse: collapse;"> <tr><td></td><td></td><td>H</td><td>T</td><td>U</td></tr> <tr><td></td><td>5</td><td>10</td><td>1</td><td></td></tr> <tr><td></td><td>6</td><td>0</td><td>5</td><td></td></tr> <tr><td></td><td>-</td><td>3</td><td>2</td><td>8</td></tr> <tr><td></td><td></td><td>2</td><td>7</td><td>7</td></tr> </table> </div>			H	T	U		5	10	1			6	0	5			-	3	2	8			2	7	7	<div style="border: 2px solid purple; border-radius: 15px; padding: 10px; background-color: #e0ffe0;"> <p>It is even possible, for children who find column method very difficult to remember, or who regularly make the same mistakes, to use the number line method for 4 digit numbers, using either of the approaches.</p> </div>
		H	T	U																							
	5	10	1																								
	6	0	5																								
	-	3	2	8																							
		2	7	7																							
<p><b>Y4</b></p>	<p>From late Y4 onwards, move onto examples using 4-digit (or larger) numbers and then onto decimal calculations.</p>	<p style="text-align: center;"><b>5042 – 1776</b></p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p><b>S9d: 1000s, 100s, 10s, 1s Jump</b></p> <table style="margin: auto; border-collapse: collapse;"> <tr><td></td><td>+3000</td><td>+200</td><td>+60</td><td>+6</td></tr> <tr><td>1776</td><td>4776</td><td>4976</td><td>5036</td><td>5042</td></tr> <tr><td colspan="5" style="text-align: center;"><b>5042 - 1776 = 3266</b></td></tr> </table> </div>		+3000	+200	+60	+6	1776	4776	4976	5036	5042	<b>5042 - 1776 = 3266</b>														
	+3000	+200	+60	+6																							
1776	4776	4976	5036	5042																							
<b>5042 - 1776 = 3266</b>																											



If necessary, apparatus can still be used to demonstrate the exchange / regroup principle.

**CPA Reasoning**  $5042 - 1776 = 3266$  Part 1

1.  $5042 - 1776$

2.  $5042 - 1776$

3.  $5042 - 1776$

4.  $5042 - 1776$

5.  $5042 - 1776$

6.  $5042 - 1776$

Sense of Number Primary School

**CPA Reasoning**  $5042 - 1776 = 3266$  Part 2

7.  $5042 - 1776$

8.  $5042 - 1776$

9.  $5042 - 1776$

10.  $5042 - 1776$

11.  $5042 - 1776$

12.  $5042 - 1776$

Sense of Number Primary School

**S1d: Column Subtraction**

$$\begin{array}{r} \overset{9}{\cancel{5}} \overset{13}{4} \overset{1}{2} \\ - 1776 \\ \hline 3266 \end{array}$$

Sense of Number Primary School

**S8d: Quad Jump Extreme**

$+24 +200 +3000 +42$

$1776 \quad 1800 \quad 2000 \quad 5000 \quad 5042$

$5042 - 1776 = 3266$

Sense of Number Primary School

**Y5/6**

In Years 5 & 6 apply to any 'big number' examples.

**S1le: Column Subtraction**

$$\begin{array}{r} \overset{9}{\cancel{7}} \overset{1}{2} \overset{7}{8} \overset{12}{3} \overset{1}{1} \\ - 427358 \\ \hline 315473 \end{array}$$

Sense of Number Primary School

Both methods can be used with decimals, although the counting up method becomes less efficient and reliable when calculating with more than two decimal places.

$13.4 - 8.7$

**S1lf: Column Subtraction**

$$\begin{array}{r} \overset{10}{\cancel{13}} \overset{1}{.} \overset{1}{4} \\ - 8.7 \\ \hline 4.7 \end{array}$$

Sense of Number Primary School

$13.4 - 8.7$

**S9f: Is Jump, Tenths Jump!**

$+4 \quad +0.7$

$8.7 \quad 12.7 \quad 13.4$

$13.4 - 8.7 = 4.7$

Sense of Number Primary School

$12.4 - 5.97$

**S1lh: Column Subtraction With Decimals**

$$12.4 - 5.97 = 6.43$$

$$\begin{array}{r} \overset{10}{\cancel{12}} \overset{1}{.} \overset{13}{4} \overset{0}{0} \\ - 5.97 \\ \hline 6.43 \end{array}$$

Sense of Number Primary School

$12.4 - 5.97$

**S8x1: Decimal T-J!**

$+0.03 \quad +6 \quad +0.4$

$5.97 \quad 6 \quad 12 \quad 12.4$

$12.4 - 5.97 = 6.43$

Sense of Number Primary School



72.43 – 47.85

Even with calculations to 2 decimal places, practical apparatus can be used initially to explore / embed understanding or at a later stage for the children to demonstrate greater depth. In these instances, they can recreate a column method with decimals and explain each stage of the procedure.

**S8x2: Decimal T-J!**

+0.15    +24    +0.43

47.85    48    72    72.43

72.43 - 47.85 = 24.58

*Sense of Number Primary School*

**CPA reasoning 72.43 - 47.85 = 24.58 Part 2**

*Sense of Number Primary School*

**CPA reasoning 72.43 - 47.85 = 24.58 Part 1**

*Sense of Number Primary School*

**S11g: Column Subtraction**

10    1    =     $\frac{1}{10}$      $\frac{1}{100}$

6    11    13    1

~~72.43~~

- 47.85

---

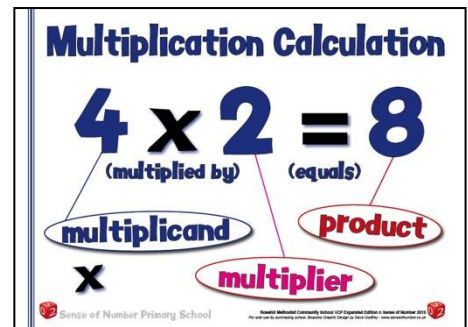
24.58

*Sense of Number Primary School*



# Multiplication Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.



These notes show the stages in building up to using an efficient method for

- two-digit by one-digit multiplication by the end of Year 3,
- three-digit by one-digit multiplication by the end of Year 4,
- four-digit by one-digit multiplication and two/three-digit by two-digit mult. by the end of Year 5
- three/four-digit by two-digit multiplication and multiplying 1-digit numbers with up to 2 decimal places by whole numbers by the end of Year 6.

To multiply successfully, children need to be able to:

- recall all multiplication facts to  $12 \times 12$ ;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as  $70 \times 5$ ,  $70 \times 50$ ,  $700 \times 5$  or  $700 \times 50$  using the related fact  $7 \times 5$  and their knowledge of place value;
- similarly apply their knowledge to simple decimal multiplications such as  $0.7 \times 5$ ,  $0.7 \times 0.5$ ,  $7 \times 0.05$ ,  $0.7 \times 50$  using the related fact  $7 \times 5$  and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as  $60 + 70$ ) or of 100 (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

## Note:

Children need to acquire **one efficient written method of calculation for multiplication, which** they know they can rely on **when mental methods are not appropriate.**

**It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.**

These mental methods are often more efficient than written methods when multiplying.

*Use partitioning and grid methods until number facts and place value are secure*

*For a calculation such as  $25 \times 24$ , a quicker method would be 'there are four 25s in 100 so  $25 \times 24 = 100 \times 6 = 600$ '*

*When multiplying a 3 / 4 digit x 2-digit number the standard method is usually the most efficient*

*At all stages, use known facts to find other facts.  
E.g. Find  $7 \times 8$  by using  $5 \times 8$  (40) and  $2 \times 8$  (16)*





# Mental Multiplication Strategies

In a similar way to addition, multiplication has a range of mental strategies that need to be developed both before and then alongside written methods (both informal and formal). Some of these are the same strategies used for addition but adapted for multiplication. Others are specifically multiplication strategies, which enable more difficult calculations to be worked out much more efficiently.

**Mental Multiplication**

- 132 **MM1** Manipulate Calculation
- 139 **MM2** Factorising
- 146 **MM3** Re-ordering
- 149 **MM4** Partitioning
- 154 **MM5** Round & Adjust
- 158 **MM6** Doubling
- 166 **MM7** Doubling Table Facts
- 170 **MM8** Doubling Up
- 173 **MM9** Multiply by ... then Halve
- 175 **MM10** Jump

**10 Cool Strategies for Mental Multiplication**

Sense of Number Primary School

## Tables Facts

In Key Stage 2, however, before any written methods can be securely understood, children need to have a bank of multiplication tables facts at their disposal, which can be recalled instantly.

The learning of tables facts does begin with counting up in different steps, but by the end of Year 4 it is expected that most children can instantly recall all facts up to 12 x 12.

The progression in facts is as follows (11's moved into Y3 as it is a much easier table to recall): -

**Y2**

1 x table

Mx2: Table Facts

**2x table**

Mx5: Table Facts

**5x table**

Mx10: Table Facts

**10x table**

**Y3**

Mx3: Table Facts

**3x table**

Mx4: Table Facts

**4x table**

Mx8: Table Facts

**8x table**

Mx11: Table Facts

**11x table**

**Y4**

Mx6: Table Facts

**6x table**

Mx7: Table Facts

**7x table**

Mx9: Table Facts

**9x table**

Mx12: Table Facts

**12x table**

Once the children have established a bank of facts, they are ready to be introduced to jottings and eventually written methods.



## Doubles & Halves

The other facts that children need to know by heart are **doubles and halves**. These are no longer mentioned explicitly within the National Curriculum, making it even more crucial that they are part of a school's mental calculation policy.

If children haven't learned to recall simple doubles instantly, and haven't been taught strategies for mental doubling, then they cannot access many of the mental calculation strategies for multiplication (E.g. Double the 4 times table to get the 8 times table. Double again for the 16 times table etc.). Halving numbers is particularly crucial when working with fractions (From 1/2s to 1/4s to 1/8s etc. Children who can halve numbers effectively can quickly work out that 1/8 of 136 is 17 by halving 3 times.

As a general guidance, children should know the following doubles & halves: -

Year Group	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<b>Doubles and Halves</b>	All doubles and halves from double 1 to double 10 / half of 2 to half of 20	All doubles and halves from double 1 to double 20 / half of 2 to half of 40 (E.g. double 17=34, half of 28 = 14)	Doubles of all numbers to 100 with units digits 5 or less, and corresponding halves (E.g. Double 43, double 72, half of 46)  Reinforce doubles & halves of all multiples of 10 & 100 (E.g. double 800, half of 140)	Addition doubles of numbers 1 to 100 (E.g. 38 + 38, 76 + 76) and their corresponding halves  Revise doubles of multiples of 10 and 100 and corresponding halves	Doubles and halves of decimals to 10 – 1 d.p. (E.g. double 3.4, half of 5.6)	Doubles and halves of decimals to 100 – 2 d.p. (E.g. double 18.45, half of 6.48)

Before certain doubles / halves can be recalled, children can use a simple jotting to help them record their steps towards working out a double / half.

### Y2

**MM6: Doubling**

$$20 + 14 = 34$$

**Double 17 = 34**

$$(15 + 2)$$

$$30 + 4 = 34$$

### Y3

**MM6a: Doubling**

$$60 + 14 = 74$$

**Double 37 = 74**

$$(35 + 2)$$

$$70 + 4 = 74$$

### Y4

**MM6b: Doubling**

$$140 + 16 = 156$$

**Double 78 = 156**

$$(75 + 3)$$

$$150 + 6 = 156$$

**MM6c: Doubling**

**Double 340 = 680**

$$600 + 80 = 680$$

### Y4/5

**MM6d: Doubling**

$$800 + 160 = 960$$

**Double 480 = 960**

$$(450 + 30)$$

$$900 + 60 = 960$$

**MM6e: Doubling**

$$400 + 140 + 16 = 556$$

**Double 278 = 556**

$$(250 + 28)$$

$$500 + 28 = 556$$

### Y5/6

**MM6f: Doubling**

$$1400 + 120 + 16 = 1536$$

**Double 768 = 1536**

$$(750 + 18)$$

$$1500 + 36 = 1536$$

**MM6g: Doubling**

**Double 3.7 = 7.4**

$$6 + 1.4 = 7.4$$



Once the children have learned their doubles and their tables facts, there are then several mental calculation strategies that need to be taught so that children can continue to begin any calculation with the question **'Can I do it in my head?'**

The majority of these strategies are usually taught in Years 4 – 6, but there is no reason why some of them cannot be taught earlier as part of the basic rules of mathematics.

## Multiplying by 10 / 100 / 1000

This strategy is usually part of the Year 5 & 6 teaching programme for decimals, namely that digits move to the left when multiplying by 10, 100 or 1000, and to the right when dividing.

This also secures place value by emphasising that the decimal point doesn't ever move, and that the digits move around the decimal point (not the other way round, as so many adults were taught at school).



It would be equally beneficial to teach a simplified version of this strategy in KS1 / Lower KS2, encouraging children to move digits into a new column, rather than simply 'adding zeroes' when multiplying by 10/100.

The 2014 Curriculum emphasises the need for children to have conceptual understanding of any procedures that they have learned, and 'zero as a place holder' can be taught very effectively with place value resources such as Base 10.

For example, multiplying 34 by 10 (the numbers seen above) would involve getting 3 Tens and 4 Ones then making each 10 times bigger.

3 Tens multiplied by 10 would give 30 Tens (which would be exchanged for 3 Hundreds)

4 Ones multiplied by 10 would give 40 Ones (which would be exchanged for 4 Tens).

The image would then be 3 Hundreds and 4 Tens or 340.

In the section on Mental Addition there were 6 key strategies outlined.

The following 4 strategies for mental multiplication can be explicitly linked to 4 of the strategies in mental addition –

## Partitioning, Round & Adjust, Re-Ordering (Number Bonds) & Manipulate The Calculation





## Partitioning

**Partitioning** is an equally valuable strategy for multiplication as it is for addition.

It can be quickly developed from a jotting to a method that is completed entirely mentally.

It is clearly linked to the grid method of multiplication, but should also be taught as a 'partition jot' so that children, by the end of Year 4, have become skilled in mentally partitioning 2 and 3 digit numbers when multiplying (with jottings when needed).

By the time children leave Year 6 they should be able to mentally partition most simple 2 & 3 digit, and also decimal multiplications, and should, wherever possible, be trying to work these out mentally.

**MM3: Partitioning**

$$15 \times 5 = 75$$

$$\begin{array}{c} \text{50} + \text{25} = 75 \\ (10 \times 5) \quad (5 \times 5) \end{array}$$

Sense of Number Primary School

**MM4c: Partitioning**

$$4.3 \times 8 = 34.4$$

$$\begin{array}{c} \text{32} + \text{2.4} = 34.4 \\ (4 \times 8) \quad (0.3 \times 8) \end{array}$$

Sense of Number Primary School

## Round & Adjust

**Round & Adjust** is also a high quality mental strategy for multiplication, especially when dealing with money problems in upper KS2. Once children are totally secure with rounding and adjusting in addition, they can be shown how the strategy extends into multiplication, where they round then adjust by the multiplier.

E.g. For  $49 \times 3$  round to  $50 \times 3$  (150) then adjust by  $1 \times 3$  (3) to give a product of  $150 - 3 = 147$ .

For  $198 \times 4$ , round to  $200 \times 4$  (800) then adjust by  $2 \times 4$  (8) for a product of  $800 - 8 = 792$

**MM4: Round & Adjust**

$$49 \times 3 = 147$$

$$\begin{array}{c} (50 \times 3) - (1 \times 3) \\ 150 - 3 = 147 \end{array}$$

Sense of Number Primary School

**MM4a: Round & Adjust**

$$198 \times 4 = 792$$

$$\begin{array}{c} (200 \times 4) - (2 \times 4) \\ 800 - 8 = 792 \end{array}$$

Sense of Number Primary School

**MM4b: Round & Adjust**

$$3.9 \times 5 = 19.5$$

$$\begin{array}{c} (4 \times 5) - (0.1 \times 5) \\ 20 - 0.5 = 19.5 \end{array}$$

Sense of Number Primary School

**MM4c: Round & Adjust**

$$£5.99 \times 6 = £35.94$$

$$\begin{array}{c} (£6 \times 6) - (1p \times 6) \\ £36 - 6p = £35.94 \end{array}$$

Sense of Number Primary School

**Y4**

**Y4/5**

**Y5**

**Y5/6**

## Re-ordering

**Re-ordering** is similar to **Number Bonds** in that the numbers are calculated in a different order. With **Number Bonds** in addition the children look for a simple bond that will make the numbers easier to add.



In **Re-ordering**, the children look at the numbers that need to be multiplied, and, using commutativity, rearrange them so that the calculation is easier to complete.

The slides below show various examples of re-ordering.


Each slide re-orders the calculation in 3 ways. The asterisked calculation in each of the examples is probably the easiest / most efficient rearrangement of the numbers. For example, when multiplying  $7 \times 4 \times 5$  it is much quicker to multiply the  $5 \times 4$  first.

**MM2: Re-ordering**

$(9 \times 2) \times 5$   
 $18 \times 5 = 90$

$(9 \times 5) \times 2$   
 $45 \times 2 = 90$

$(2 \times 5) \times 9$   
 $10 \times 9 = 90$  \*




**MM2a: Re-ordering**

$(7 \times 4) \times 5$   
 $28 \times 5 = 140$

$(7 \times 5) \times 4$   
 $35 \times 4 = 140$

$(4 \times 5) \times 7$   
 $20 \times 7 = 140$  \*




**MM2b: Re-ordering**

$(9 \times 8) \times 6$   
 $72 \times 6 = 432$

$(9 \times 6) \times 8$   
 $54 \times 8 = 432$  \*

$(8 \times 6) \times 9$   
 $48 \times 9 = 432$



## Manipulate Calculation

In addition, when applying this strategy, the children 'passed over' part of one number to the other in order to simplify the calculation.

In multiplication, however, they look at the numbers involved and determine whether an easier calculation can be created by dividing one of the numbers by a chosen amount and then multiplying the other by the same amount

This is probably the best strategy available for simplifying a calculation quickly, as long as the numbers being multiplied are appropriate.

To develop understanding of this strategy in Key Stage 1 or lower Key Stage 2, children can use practical apparatus to create an array. (E.g.  $8 \times 3$ )

They can then split the array in half, moving one half over the top of the other. This creates a new array of  $4 \times 6$ .

If they repeat this again they can create an array of  $2 \times 12$ . If they repeat it a final time then the array would be  $1 \times 24$ .

This demonstrates the general principle that if you double one number within a multiplication, and halve the other number, then the product stays the same.

Once this has been proved, the strategy can be developed to show that if you treble / quadruple one number within a multiplication, and third / quarter the other number, then the product will still stay the same. This can then be generalised for any multiple / Fraction.

E.g.  $27 \times 3$  is a 2 digit by 1 digit calculation. It can be manipulated into a known fact ( $9 \times 9$ ) if the **27 is divided by 3** and the **3 is multiplied by 3**.



$26 \times 32$  is quite a difficult long multiplication but if we **multiply the 26 by 4** (by doubling twice) and **divide the 32 by 4** we manipulate the calculation into a far easier  $104 \times 8$ .

**MM1a: Manipulate Calculation**  
5

$27 \times 3$

$+3$     $\times 3$

$9 \times 9 = 81$

Sense of Number Primary School

**MM1e: Manipulate Calculation**  
6

$26 \times 32$

$\times 4$     $\div 4$

$104 \times 8 = 832$

Sense of Number Primary School

**Doubling** strategies are probably the most crucial of the mental strategies for multiplication, as they can make difficult long multiplication calculations considerably simpler.

### Doubling Tables Facts

Initially, children are taught to double one table to find another.

E.g. Doubling the 4s to get the 8s (Eg 1 below)

If  $4 \times 6 = 24$  then  $8 \times 6$  must be 48 because 8 is double 4 ( $4 \times 12$  is also 48 as 12 is double 6)

This can then be applied to any table: -

If  $8 \times 7 = 56$  then  $16 \times 7$  must be 112 because 16 is double 8 (or  $8 \times 14 = 112$ )

If  $11 \times 12 = 132$  then  $22 \times 12$  must be 264 because 22 is double 11 (or  $11 \times 24 = 264$ )

**MM7: Doubling Table Facts**  
3

$8 \times 6 = 48$   
( $4 \times 2$ )

$4 \times 6 = 24$

$\downarrow$     $\downarrow$     $\times 2$

$8 \times 6 = 48$

Sense of Number Primary School

**MM7b: Doubling Table Facts**  
5

$16 \times 7 = 112$   
( $8 \times 2$ )

$8 \times 7 = 56$

$\downarrow$     $\downarrow$     $\times 2$

$16 \times 7 = 112$

Sense of Number Primary School

**MM7c: Doubling Table Facts**  
6

$22 \times 12 = 264$   
( $11 \times 2$ )

$11 \times 12 = 132$

$\downarrow$     $\downarrow$     $\times 2$

$22 \times 12 = 264$

Sense of Number Primary School

### Doubling Up

This strategy develops the above strategy further, enabling multiples of **4**, **8** and **16** onwards to be calculated by constant doubling.

It is linked very closely to the mental division 'halving' strategy, where we can divide by 8 by halving three times.: -

Multiply by 4 – double twice    Multiply by 8 – double 3 times    Multiply by 16 – Double 4 times





**MM8: Doubling Up**  
<sup>3/4</sup>  
 $17 \times 4 = 68$

Double 17 = 34 (17 x 2)  
 Double 34 = 68 (17 x 4)

Sense of Number Primary School

**MM8a: Doubling Up**  
<sup>5</sup>  
 $36 \times 8 = 288$

Double 36 = 72 (36 x 2)  
 Double 72 = 144 (36 x 4)  
 Double 144 = 288 (36 x 8)

Sense of Number Primary School

**MM8b: Doubling Up**  
<sup>6</sup>  
 $125 \times 16 = 2000$

Double 125 = 250 (125 x 2)  
 Double 250 = 500 (125 x 4)  
 Double 500 = 1000 (125 x 8)  
 Double 1000 = 2000 (125 x 16)

Sense of Number Primary School

## Multiplying by 10 / 100 / 1000 then halving / quartering

The final doubling / halving strategy works on the principle that multiplying by 10 / 100 is straightforward, and this can enable you to easily multiply by 5, 50 or 25.

Because 5 is half of 10 you can multiply a number by 10 then half it.

E.g.  $86 \times 10 = 860$  so  $86 \times 5$  must be 430.

In a similar way,  $56 \times 100 = 5600$  so  $56 \times 25$  must be a quarter of that amount (1400)

**MM9: Mult by 10,100 then Halve**  
<sup>5</sup>

$86 \times 5 = 430$

$86 \times 10 = 860$   
 $860 \div 2 = 430$

Sense of Number Primary School

**MM9a: Mult by 100,1000 then Halve**  
<sup>6</sup>

$56 \times 25 = 1400$

$56 \times 100 = 5600$   
 $5600 \div 2 = 2800$   
 $2800 \div 2 = 1400$

Sense of Number Primary School

## Factorising

The final mental strategy within the policy is factorising.

If children use their tables knowledge they can re-write a calculation to simplify it.

$15 = 5 \times 3$  so multiplying  $32 \times 15$  can be simplified as  $32 \times (5 \times 3)$ .  $32 \times 5 = 160$  and  $160 \times 3 = 480$

Multiplying a 2-digit number by 24, for example, may be easier if multiplying by a factor pair of 24.

$52 \times 24 = 52 \times (4 \times 6)$   $52 \times 4$  is a simple 208 then  $208 \times 6$  is also a relatively straightforward 1248

This calculation could also be factorised as  $52 \times (8 \times 3)$ .  $52 \times 8 = 416$ .  $416 \times 3 = 1248$

**MM2d: Factorising**  
<sup>6</sup>

$32 \times 15 = 480$

$(32 \times 5 \times 3)$

$160 \times 3 = 480$

Sense of Number Primary School

**MM2c: Factorising**  
<sup>5/6</sup>

$36 \times 25 = 900$

$(9 \times 4 \times 25)$

$9 \times 100 = 900$

Sense of Number Primary School

**MM2e: Factorising**  
<sup>6</sup>

$26 \times 32 = 832$

$(26 \times 4 \times 8)$

$104 \times 8 = 832$

Sense of Number Primary School



# Models of Multiplication

**CPA** Models of **Multiplication**

reasoning

## Repeated Addition

$5 + 5 + 5 = 15$

“I can fit 5 footballs in a bag and I’ve brought 3 full bags. How many footballs do I have with me?” “15”

## Scaling

“My tower of football stickers was 5 high, but now it’s 3 times as big! How many stickers in my tower?” “15”

$5 \times 3 = 15$

A.N. Other Primary School

A.N. Other Primary School Concrete & Pictorial VCP © Sense of Number 2017  
For sole use by purchasing school. Bespoke Graphic Design by Dave Godfrey - www.senseofnumber.co.uk

The poster above shows the two images for multiplication which are used within the Visual Calculation Policy, either **repeated addition** or **scaling**, The reasons for adopting these models are explained below: -

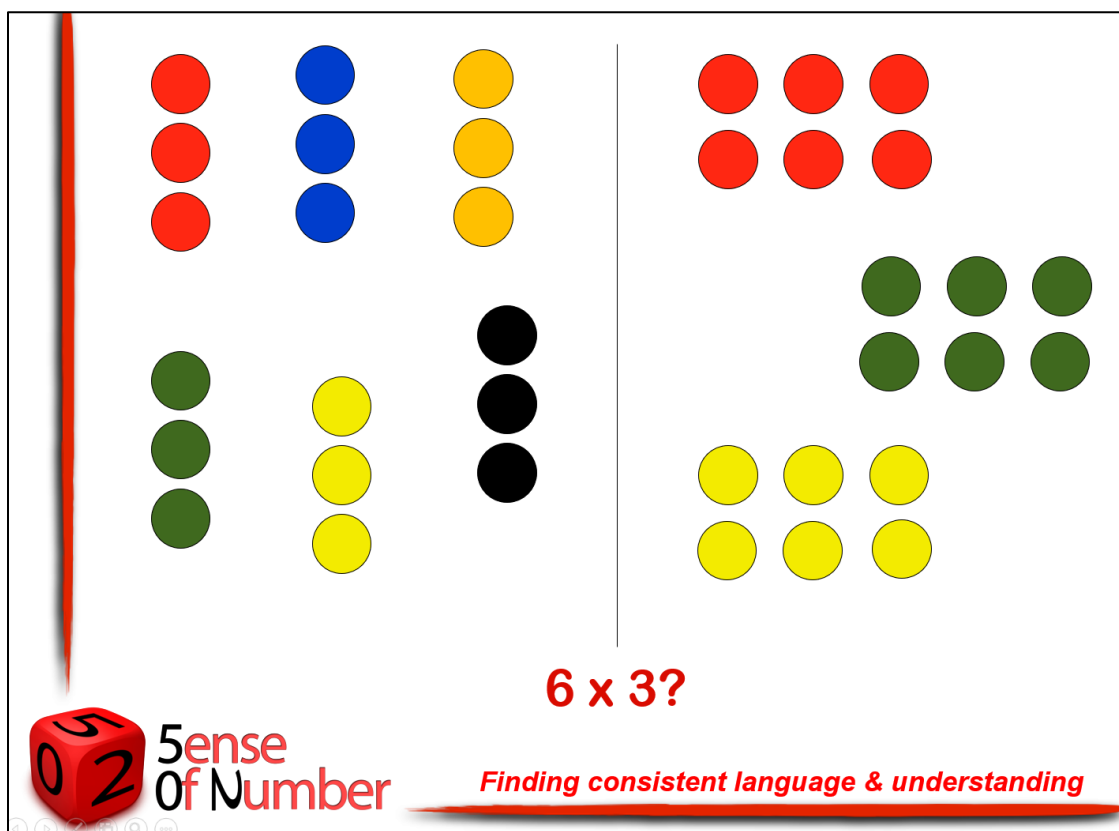
## Understanding Multiplication

One of the most difficult aspects when teaching multiplication is to ensure that it is taught consistently across the school. Due to the way that most teachers were themselves taught multiplication, there tend to be two approaches adopted across the UK.

For a calculation such as  $6 \times 3$ , it is commonly viewed either as ‘6 lots of 3’ (see left hand side of image below) or ‘6 repeated three times’ (see right hand image).

Teachers in KS1 sometimes move between the two images without realising, but usually settle on one particular explanation.

In later years, however, the image / explanation taught in KS1 often conflicts with the understanding needed to visualise a higher level calculation in KS2. Therefore, it is important that the image and explanation introduced in KS1 is the same image / explanation which continues to be used and developed in KS2.



## ‘Lots of’ or ‘Repeat a number of times’???

It can be argued that both images above are models of  $6 \times 3$ , depending on your own interpretation, but only one image is mathematically accurate if we are teaching **multiplication**.

The key question to ask is ‘Which image is actually showing 6 multiplied by 3?’ In effect, which image takes 6 then multiplies it to make it 3 times as big?

This means that the correct image is the **second one**.

If we use the correct vocabulary and regularly say ‘**multiplied by**’ when reading a multiplication calculation then the second image would be the one which would automatically come to mind.

For example, ‘**10 multiplied by 4**’ would mean **10 objects (fingers on hands) repeated 4 times**.

Teaching children that  $6 \times 3$  is ‘6 lots of 3’ does give the same answer of 18, and when displayed as an array can be read either way, but saying that ‘times means lots of’ is not mathematically accurate and means that you are unable to use the word multiply correctly.

Consequently, this policy is consistent throughout in its use of ‘repeated addition’ and ‘multiply by’. In the example above,  $6 \times 3$  means ‘take 6 objects then multiply the 6 objects by 3’, showing us ‘**6 three times**’



## Scaling

The other way in which  $6 \times 3$  can be explained correctly is through the use of **scaling**.

$6 \times 3$  means 6 scaled up to be '**3 times as big**'.

This interpretation is commonly used throughout Europe and means that children see a multiplication calculation and immediately picture the answer as something which has been scaled up or down in size.

For example,  $8 \times 5$  could be used for a word problem such as '*John has an 8cm piece of string. Louise has a piece of string which is 5 times as long. How long is Louise's string?*'

In this instance there is no repeated addition **or** 'lots of'. Louise's string isn't 8cm repeated 5 times (although that would give the same answer if they were laid out end to end) and it certainly isn't 8 lots of 5 cm. It is a single 40cm piece of string which is 5 times longer than John's 8cm piece.

Scaling is also very helpful for multiplying by fractions.  $12 \times \frac{1}{3}$  would mean 12 scaled to be  $\frac{1}{3}$  as big (i.e. 3 times smaller)  $\frac{1}{3} \times 12$  would be  $\frac{1}{3}$  made 12 times as big or  $\frac{1}{3}$  repeated 12 times. Both methods would give the correct answer of 4.

This is why the poster shows the two images for multiplication used within the Visual Calculation Policy as either **repeated addition** or **scaling**, but not 'lots of'.

To summarise, here are the key reasons why 'lots of', although giving the correct answer, is not mathematically accurate: -

## Conclusion: Why is $6 \times 3$ not really '6 lots of 3'?

1. If we use the word **multiply** we would take 6 then **multiply it** by 3 - i.e. Get 6 ( **$6 \times 1$** ) then another 6 ( **$6 \times 2$** ) then another 6 ( **$6 \times 3$** )
2. Multiplying can also be interpreted as **scaling** –  $6 \times 3$  would be 6 made 3 times bigger (or scaled up 3 times)
3. The 'x' sign is an '**operator**' – it 'operates' or does something to the first number: -
  - **$6 + 3$  - you have 6 and then add more**
  - **$6 - 3$  - you have 6 and then remove 3 or find the difference**
  - **$6 \div 3$  - you have 6 then share it between 3 or group it into 3s**

Therefore

- **$6 \times 3$  - you would have 6 and then get 2 more sixes i.e. you wouldn't suddenly decide to pick up 3s**
4. If we ask the question 'What is another way to say ' **$\times 2$** ' the usual response is '**double**'. If we then ask the question 'What is another way to say ' **$\times 3$** ' the usual response is '**triple**' or '**treble**'.

Therefore... if we take 6 then **times by 3** we are **trebling** the 6.



# Written Methods Of Multiplication

Stage 1	Number Lines, Arrays & Mental Methods
<p><b>FS</b></p>	<p>In Early Years, children are introduced to grouping, and are given regular opportunities to put natural resources and real life objects into groups of 2, 3, 4, 5 and 10.</p> <p>They also stand in different sized groups, and use the term 'pairs' to represent groups of 2.</p> <p>This is then developed into using resources such as Base 10 apparatus, Numicon, multi-link or an abacus. Children begin to visualise counting in ones, twos, fives and tens, saying the multiples as they count the pieces.</p> <p>E.g. Saying '10, 20, 30' or 'Ten, 2 tens, 3 tens' whilst counting Base 10 pieces</p>
<p><b>Y1</b></p>	<div data-bbox="368 539 730 792" data-label="Image"> </div> <p>Begin by introducing the concept of multiplication as repeated addition. Before using mathematical apparatus, use real objects and equipment such as cups, cakes, footballs, pencils, apples etc.)</p> <p>Children will firstly make then draw these objects in groups giving the product by counting up in 2s, 5s, 10s and beyond, and finally by writing the multiplication statement.</p> <p>The picture above will begin as an addition 'story' (<b>5 footballs and 5 footballs makes 10 footballs</b>) but will then be written as a multiplication calculation (<b><math>5 \times 2 = 10</math></b>)</p> <p><b>Make sure from the start</b> (as explained in the introduction to this section) <b>that all children say the multiplication fact the correct way round, using the word 'multiply' more often than the word 'times' so that they understand what 'multiplication' means'</b></p> <p>For the example above, there are 5 footballs in 2 groups, showing <b>5 multiplied by 2 (<math>5 \times 2</math>)</b>, not 2 times 5. It is the <b>'5'</b> which is being repeatedly added / <b>scaled up</b> / made bigger / <b>multiplied</b>. <b>'5 multiplied by 2'</b> shows <b>'2 groups of 5' or 'Two fives'</b></p> <p>Again, using resources and real life objects, involve the children in telling stories and creating pictures for the 3s and 4s, and then into writing multiplication statements.</p> <p>E.g. <b>3 footballs + 3 footballs + 3 footballs + 3 footballs would show <math>3 \times 4 = 12</math> footballs</b></p>
<p><b>Y2</b></p>	<p style="text-align: center;"><b>The array</b></p> <div data-bbox="400 1317 863 1644" data-label="Image"> </div> <div data-bbox="895 1317 1347 1653" data-label="Image"> </div> <p>Build on children's understanding that multiplication is repeated addition, using arrays and number lines to support their thinking.</p> <p>Start to develop the use of the array to show linked facts (commutativity).</p> <p>Make arrays with a wide range of objects, especially those which naturally occur in real-life such as windows, egg boxes, drawers or cake trays.</p> <p>Emphasise that all multiplications can be worked out either way (<math>2 \times 5 = 5 \times 2 = 10</math>) as this will support the children in the future learning of their tables facts.</p> <p>Practice counting in both steps (E.g. Both 2s and 5s) to prove that the answer is the same no matter which way round they are counted.</p> <p>Encourage the children to see how the array makes it easier to see the calculation as a multiplication rather than a repeated addition.</p>

As with addition & subtraction, the key to developing clear understanding of multiplication is to involve the children in visualizing calculations with a wide range of resources. Using manipulatives such as Numicon, abacuses, number rods & multi-link, children can demonstrate & discuss their interpretation of multiplication statements

**Continue to emphasise multiplication the correct way round.**  
**E.g.  $5 \times 3 = 5 + 5 + 5$**   
 **$5$  multiplied by  $3 = 15$**

The above 'materials can then be used alongside simple pictures and jottings which support the transition from repeated addition to multiplication.

Extend the above to include the 3, 4, 8 and 11 times tables (Year 3) then the 6, 7, 9 & 12 times tables (Year 4)

**Y3**  
**&**  
**Y4**

Even when dealing with larger tables such as the 7's and the 8's it is crucial that children can create a model or image of the calculation.

The footballs simply show four 7s and nine 8's as a real life picture, but the other images support children in either seeing the product or understanding the calculation in more depth.

The **Multi-link** and **Abacus** images are especially useful as they allow the **7 times table** to be seen as a combination of the **5s and 2s** and the **8 times table** as **5s and 3s**.

E.g. the 7 x 4 multi-link / abacus images are colour-coded to show a 5x4 array and a 2x4 array. The 8 x 9 images show 3x9 and 5x9.

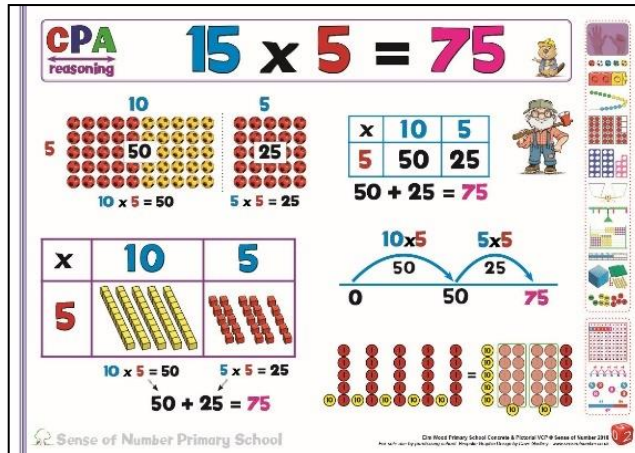
The Number Rod image places the four 7s / nine 8s on a track to display the products 28 / 72, whilst the Numicon pieces are arranged on a Numicon 'track' to show the same product.

If the children become accustomed to using resources to support their counting, then use similar resources when displaying a visual for multiplication, they will automatically feel more confident when asked to picture / create / visualize a higher level calculation such as those later in this policy.





Extend the use of resources for 2 digit x 1 digit calculations so that children can visualize what the calculation looks like before they are taught any specific written methods or jottings.



In each of the images above,  $15 \times 5$  can be shown as a basic Tens and Ones partition (i.e.  $10 \times 5$  and  $5 \times 5$ ) but the images allow different visualisations.

The **footballs** are laid out like a **Slavonic abacus**, allowing a clear visual of all 75 footballs but in an arrangement which makes them very easy to calculate.

The **Base 10 apparatus** is probably the most important image, and shows how the Tens and Ones are actually **partitioned** within a Grid Method – **5 Tens and 5 Ones**.

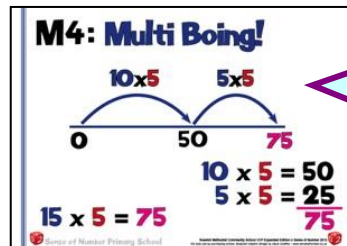
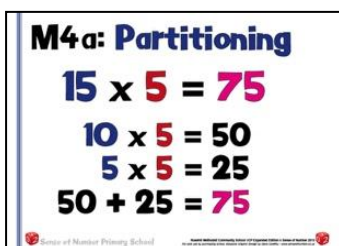
This is the model (along with the Place Value Counters) which the children need to 'make' most often in class so that they become accustomed to exchanging / regrouping Ones into Tens (and then Tens into Hundreds with more complex calculations)

The **Place Value Counters** then demonstrates how the Tens and Ones are **regrouped**. **Twenty** of the **Ones** are exchanged / regrouped into **2 Tens**, giving **7 Tens and 5 Ones**.

The Grid Method (covered in detail later) and number line (see below) are also pictured.

**Y3**

Once the general models and images for multiplication are secure, begin to partition the 2 digit number using **jottings and number lines**.



*Each of these methods can be used in the future if children find expanded or standard methods difficult.*

Extend the methods above to calculations which give products greater than 100.

## Use of 'Grid' Method within the New Curriculum

In the New Curriculum (2014), the Grid Method is not exemplified as a written method for multiplication. The only methods specifically mentioned are column procedures.

Most schools in the UK, however, have effectively built up the use of the grid method over the past 15 years, to the extent that it is generally accepted as one of the most appropriate 'written' methods for simple 2 and 3 digit x single digit calculations (**short multiplication**).

It develops clear understanding of place value through partitioning as well as being an efficient method, and is especially useful in Years 4 and 5.

Consequently, grid method is a key element of this policy, but, to align with the New Curriculum, is classed as a mental 'jotting'. It builds on partitioning, and is also the key mental multiplication method used by children in KS2 (pg 44 – multiplication partitioning)

**M5: Grid Method**  
Short Multiplication

$$15 \times 5 = 75$$

x	10	5
5	50	25

$$50 + 25 = 75$$

**M5a: Grid Method**  
Short Multiplication

$$43 \times 6 = 258$$

x	40	3
6	240	18

$$240 + 18 = 258$$

**M5b: Grid Method**  
Short Multiplication

$$147 \times 4 = 588$$

x	100	40	7
4	400	160	28

$$400 + 160 + 28 = 588$$

The examples above show the development of grid method from a straightforward 2 digit x 1 digit calculation with a product below 100 ( $15 \times 5$ ) to a more complex version ( $43 \times 6$ ) and then a 3 digit x 1 digit calculation ( $147 \times 4$ ). In each example the partitioning / place value link is very clear.

The Grid Method is equally as efficient for 2 digit x 2 digit calculations, providing an excellent basis and security for long multiplication.

Many children who find the column procedure difficult when faced with long multiplication are far more successful with the Grid Method, which builds on all of the place value and partitioning work developed earlier.

Column procedures still retain some element of place value, but, particularly for long multiplication, tend to rely on memorising a 'method', and can lead to many children making errors with the method (which order to multiply the digits, when to 'add the zero', dealing with the 'carry' digits' etc.) rather than the actual calculation. In these instances children will continue to use the grid method.

Once the calculations become more unwieldy (4 digit x 1 digit or 3 / 4 digit x 2 digit) then grid method begins to lose its effectiveness, as there are too many zeroes and part products to deal with.

At this stage column procedures are far easier, and, once learned, can be applied much quicker.

**M8: Grid Method**  
Long Multiplication

$$43 \times 65 = 2795$$

x	40	3
60	2400	180
5	200	15

$$2400 + 180 + 200 + 15 = 2795$$

**M9: Long Multiplication**  
Column

$$\begin{array}{r} 43 \\ \times 65 \\ \hline 215 \quad (5 \times 43) \\ + 2580 \quad (60 \times 43) \\ \hline 2795 \end{array}$$

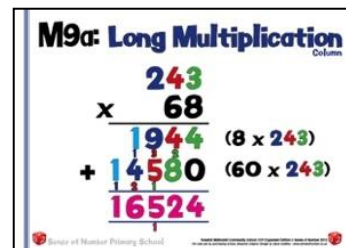
**M8a: Grid Method**  
Long Multiplication

$$243 \times 68 = 16,524$$

x	200	40	3
60	12000	2400	180 = 14,580
8	1600	320	24 = 1,944

$$14580 + 1944 = 16,524$$


Grid methods can still be used by some pupils who find columns difficult to remember, and who regularly make errors, but children should be encouraged to move towards columns for more complex calculations



Stage 2 Written Methods - Short Multiplication		
	Grid Multiplication (Mental 'Jotting')	Column multiplication (Expanded method into standard)
Y3	<p>The <b>grid method</b> of multiplication is a simple, alternative way of recording the partitioning jottings shown previously. <i>As shown earlier, it can initially be taught using an array to show the actual product</i></p>	<p>The expanded method links the grid method to the standard method. It still relies on partitioning the tens and units, but sets out the products vertically.</p>
	<p>It is recommended that the grid method is used as the main method within Year 3. It clearly maintains place value, and helps children to visualise and understand the calculation better.</p>	<p>Children will use the expanded method until they can securely use and explain the standard method.</p>
		<p>At some point within the year, the column method can be introduced, and children given the choice of using either grid or standard. Some schools may delay the introduction of column method until Year 4</p> <p><i>Place the 'carry' digit below the line</i></p> <p><i>When setting out calculations vertically, begin with the ones first (as with addition and subtraction).</i></p>



**CPA reasoning**  $43 \times 6 = 258$

x	40	3
6		

$40 \times 6 = 240$     $3 \times 6 = 18$

$240 + 18 = 258$

**258**

**240 + 18 = 258**

Sense of Number Primary School

it is important to continue adopting the CPA approach by making and explaining the more complex 2 digit x 1 digit calculations.

E.g. Using Base 10 apparatus, children can demonstrate  $43 \times 6$  as 4 Tens repeated 6 times and 3 Ones repeated 6 times.

They can actually display this image within a Grid Method and then exchange / regroup 20 of the Tens into 2 Hundreds.

Continue to use apparatus regularly to support the place value and conceptual understanding of the calculation.

**Y4**

**M5a: Grid Method**  
Short Multiplication

$43 \times 6 = 258$

x	40	3
6	240	18

**240 + 18 = 258**

Sense of Number Primary School

**(M6: Expanded Column)**  
Additional 1

	100	10	1	
x	4	3		
			6	
			18	(6 x 3)
		240		(6 x 40)
		258		

Sense of Number Primary School

**(M7: Column Multiplication)**  
Additional 1

	100	10	1	
	4	3		
x			6	
			18	
		240		
		258		

Sense of Number Primary School

Continue to use **both grid and column methods** in Year 4 for more difficult **2 digit x 1 digit** calculations, extending the use of the grid method into mental partitioning for those children who can use the method this way and can simply jot down the sub products and answer.

At this point, the expanded method can still be used when necessary (to help 'bridge' grid with column), but children should be encouraged to use their favoured method (grid or column) whenever possible.

**CPA reasoning**  $147 \times 4 = 588$

x	100	40	7
4			

$400 + 160 + 28 = 588$

Sense of Number Primary School

Using **apparatus** for **3 digit x 1 digit** calculations is an excellent way to continue developing the conceptual understanding of what the calculations look like and the actual size of the number that is being manipulated.

In the example displayed, children can create **100 x 4**, **40 x 4** and **7 x 4** using Base 10 then show how the **16 Tens** can be exchanged / regrouped into **1 Hundred and 6 Tens**.

For 3 digit x 1 digit calculations, both grid and standard methods are efficient. Continue to use the grid method to aid place value and mental arithmetic. Use column method for speed, and to make the transition to long multiplication easier. If both methods are taught consistently then children in Year 4 will have a clear choice of 2 secure methods, and will be able to develop both accuracy and speed in multiplication.

**M5b: Grid Method**  
Short Multiplication

$147 \times 4 = 588$

x	100	40	7
4	400	160	28

**400 + 160 + 28 = 588**

Sense of Number Primary School

**M6b: Grid Method**  
Short Multiplication

$147 \times 4 = 588$

x	100	40	7	400
4	400	160	28	160
				+ 28
				588

Sense of Number Primary School

**M6: Expanded Column**

	100	10	1	
	1	4	7	
x			4	
			28	(4 x 7)
		160		(4 x 40)
		400		(4 x 100)
		588		

Sense of Number Primary School

**M7: Column Multiplication**

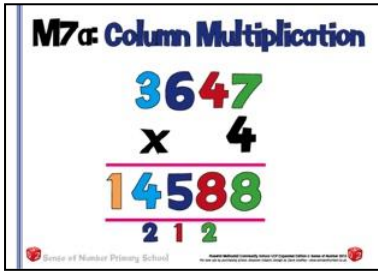
	100	10	1	
	1	4	7	
x			4	
			28	
		160		
		400		
		588		

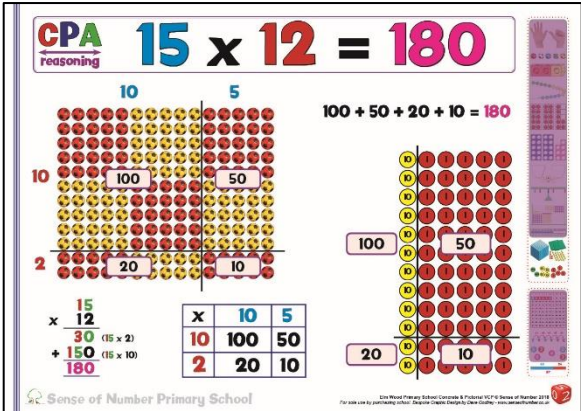
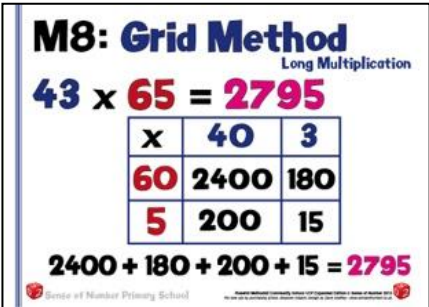
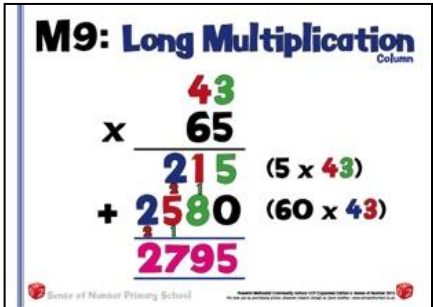
Sense of Number Primary School

Sometimes children find the multiplication and place value parts of Grid Method to be fairly

Expanded method can still be used for children who need extra support with place value (and



	<p>simple, but then struggle with the actual addition at the end (see 1<sup>st</sup> example above).</p> <p>In these instances, encourage them to complete the addition using a column method (see 2<sup>nd</sup> example above)</p>	<p>as a 'bridging' method between grid and column procedures.</p> <p>Once this is secure then they can practice the speed and security of the column method, but ensuring they can still explain the place value (using apparatus if necessary) when required</p>
<b>Y5</b>	<p>For a 4 digit x 1 digit calculation, the column method, once mastered, is quicker and less prone to error.</p> <p>The grid method may continue to be the main method used by children who find it difficult to remember the column procedure, or children who need the visual link to place value.</p>	

Stage 3		Long Multiplication (TU x TU)	
Grid Multiplication		Column multiplication (Expanded method into standard)	
		<p>Even a simple long multiplication calculation can be displayed visually using an array set out in a Slavonic Abacus arrangement. The image on the poster shows a straightforward way to quickly see all 180 counters</p> <p>At this stage, using Place Value Counters rather than Base 10 enables the calculation to be created more efficiently.</p>	
<b>Y5</b>	<p>Extend the grid method to TU x TU, asking children to estimate first so that they have a general idea of the answer. (<math>43 \times 65</math> is approximately <math>40 \times 70 = 2800</math>.)</p>  <p>As mentioned earlier, grid method is often the 'choice' of many children in Years 5 and 6, due to its ease in both procedure and understanding / place value and is the method that they will mainly use for simple long multiplication calculations.</p>	<p><b>Children should only use 'standard' column method for long multiplication if they can regularly get it correct using this method.</b></p>  <p>There is no 'rule' regarding the position of the 'carry' digits. Each choice has advantages and complications.</p> <p>Either carry the digits mentally or have your own favoured position for these digits.</p>	

# Y6

Again, estimate first:  
 $243 \times 68$  is approximately  
 $200 \times 70 = 14000$ .

**M8a: Grid Method**  
 Long Multiplication

$243 \times 68 = 16,524$

x	200	40	3	
60	12000	2400	180	= 14,580
8	1600	320	24	= 1,944
	<b>14580 + 1944 = 16,524</b>			

For 3 digit x 2 digit calculations, grid method is quite inefficient, and has much scope for error due to the number of 'part-products' that need to be added.

Use this method when you find the standard method to be unreliable or difficult to remember.

**M9a: Long Multiplication**  
 Column

$243 \times 68 = 16,524$

$8 \times 243 = 1944$   
 $60 \times 243 = 14580$

**16524**

Most children, at this point, should be encouraged to choose the standard method. For 3 digit x 2 digit calculations it is especially efficient, and less prone to errors when mastered.

Although they may find the grid method easier to apply, it is much slower / less efficient.

**M8b: Grid Method**  
 Long Multiplication

$203 \times 68 = 13,804$

x	200	0	3	
60	12000	0	180	= 12,180
8	1600	0	24	= 1,624
	<b>12180 + 1624 = 13,804</b>			

The Grid Method is an interesting way to show the part products that are created when multiplying with a 3 digit number with no Tens (or Zero in the Tens place). The 4 parts are either very large (12000&1600) or quite small (180&24). As the method shows 6 parts altogether, it is clear that 2 of them are not used.

**M9b: Long Multiplication**  
 Column

$203 \times 68 = 13,804$

$8 \times 203 = 1624$   
 $60 \times 203 = 12180$

**13804**

The column method shows the same overall product as the Grid Method (13,804) It doesn't, however, appear to be any different to a regular 3 digit x 2 digit calculation as the display simply shows the 2 part products created when multiplying by 68

**CPA Reasoning**  $3.6 \times 4 = 14.4$

Visual representation using place value counters and a grid for  $3.6 \times 4 = 14.4$ .

x	3	0.6
4	12	2.4
	<b>12 + 2.4 = 14.4</b>	

Even when multiplying decimals, Place Value Counters can be used first in order to visualise the calculation before progressing onto Grid and Column Methods

**M8c: Decimal Grid**  
 Short Multiplication

$3.6 \times 4 = 14.4$

x	3	0.6
4	12	2.4
	<b>12 + 2.4 = 14.4</b>	

Many children will find the use of Grid method as an efficient method for multiplying decimals. They must also practice mental partitioning for decimal calculations such as the one above.

**M9c: Column Multiplication**

$3.6 \times 4 = 14.4$

Column method for  $3.6 \times 4 = 14.4$ .

Extend the use of the column method into decimal multiplication.

It is advisable to set out the columns as shown on the example above so that the place value remains secure.

It is also helpful to the children in showing how many digits will need to be displayed after the decimal point.





**CPA reasoning**  $47.2 \times 3 = 141.6$

Sense of Number Primary School

**CPA reasoning**  $7.38 \times 6 = 44.28$

Sense of Number Primary School

Even when the calculations become quite complex, Place Value Counters allow an instant visualise not only of the actual calculation but also of the exchanging / regrouping which needs to take place. Ask the children to make the calculation and explain the procedure before moving to column method

**Y6**

**M8d: Decimal Grid**  
Short Multiplication  
 $47.2 \times 3 = 141.6$

x	40	7	0.2
3	120	21	0.6

$120 + 21 + 0.6 = 141.6$

Sense of Number Primary School

**M9d: Column Multiplication**

100 10 1  $\cdot \frac{1}{10}$

$$\begin{array}{r} 47.2 \\ \times 3 \\ \hline 141.6 \\ 2 \end{array}$$

Sense of Number Primary School

**M8e: Grid Method**  
Short Multiplication  
 $7.38 \times 6 = 44.28$

x	7	0.3	0.08
6	42	1.8	0.48

$42 + 1.8 + 0.48 = 44.28$

Sense of Number Primary School

**M9e: Column Multiplication**

10 1  $\cdot \frac{1}{10} \frac{1}{100}$

$$\begin{array}{r} 7.38 \\ \times 6 \\ \hline 44.28 \\ 4 \quad 2 \quad 4 \end{array}$$

Sense of Number Primary School

In the examples above, continue to think carefully about the layout of the calculation, keeping the place value accurate when multiplying.

**M8f: Grid Method**  
Long Multiplication  
 $24.3 \times 2.5 = 60.75$

x	20	4	0.3
2	40	8	0.6
0.5	10	2	0.15

$48.6 + 12.15 = 60.75$

Sense of Number Primary School

**M9f: Long Multiplication**  
Column Decimals

$$\begin{array}{r} 24.3 \\ \times 2.5 \\ \hline 12.15 \quad (0.5 \times 24.3) \\ + 48.60 \quad (2 \times 24.3) \\ \hline 60.75 \end{array}$$

Sense of Number Primary School

At this point children can use either standard method or grid method, but the column procedue tends to be more efficient.

**M9g: Long Multiplication**  
Column

$$\begin{array}{r} 3786 \\ \times 48 \\ \hline 30288 \quad (8 \times 3786) \\ + 151440 \quad (40 \times 3786) \\ \hline 181728 \end{array}$$

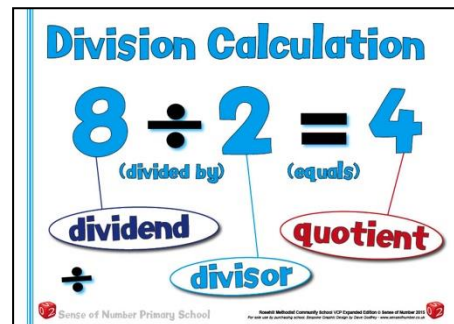
Sense of Number Primary School

By the time children meet 4 digits by 2 digits, the only efficient method is the standard method for Long Multiplication.

# Division Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to long division through Years 3 to 6 – first using short division 2 digits  $\div$  1 digit, extending to 3 / 4 digits  $\div$  1 digit, then long division 4 / 5 digits  $\div$  2 digits.



To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in  $18 \div 3 = 6$ , the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to  $12 \times 12$ , recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Children need to acquire **one efficient written method of calculation for division**, which they know they can rely on **when mental methods are not appropriate**.

**Note:** It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out expanded and standard written methods of division successfully, children also need to be able to:

- visualise how to calculate the quotient by visualising repeated addition;
- estimate how many times one number divides into another – for example, approximately how many sixes there are in 99, or how many 23s there are in 100;
- multiply a two-digit number by a single-digit number mentally;
- understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10. (e.g.  $4 \times 7 = 28$  so  $4 \times 70 = 280$  or  $40 \times 7 = 280$  or  $4 \times 700 = 2800$ .)
- subtract numbers using the column method (if using NNS 'chunking')

*For example, without a clear understanding that 72 can be partitioned into 60 and 12, 40 and 32 or 30 and 42 (as well as 70 and 2), it would be difficult to divide 72 by 6, 4 or 3 using the 'chunking' method.*

*$72 \div 6$  'chunks' into 60 and 12*


*$72 \div 4$  'chunks' into 40 and 32*

*$72 \div 3$  'chunks' into 30 and 42 (or 30, 30 and 12)*

*The above points are crucial. If children do not have a secure understanding of these prior-learning objectives then they are unlikely to divide with confidence or success, especially when attempting the 'chunking' method of division.*


Please note that there are two different 'policies' for chunking. At Boarshaw Primary School we have made the decision to teach chunking as a mental arithmetic / number line process.

## Models of Division




**CPA**  
reasoning

# Models of Division



## Sharing




**5 in each group!**

$15 \div 3 = 5$

**"If I shared my 15 footballs fairly into 3 bags, how many balls would be in each bag?" "5"**


## Grouping



**5 groups**


$15 \div 3 = 5$

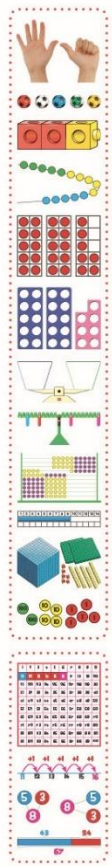
**"If I can put my 15 footballs into groups of 3, how many groups would I create?" "5"**



A.N. Other Primary School

A.N. Other Primary School Concrete & Pictorial VCP © Sense of Number 2017  
For sole use by purchasing school. Bespoke Graphic Design by Dave Godfrey - www.senseofnumber.co.uk





## Division In Key Stage 1 – Grouping or Sharing?

When children think conceptually about division, their default understanding should be **Division is Grouping, as this is the most efficient way to divide.**

The 'traditional' approach to the introduction of division in KS1 is to begin with 'sharing', as this is seen to be more 'natural' and easier to understand.

Most children then spend the majority of their time 'sharing' counters and other resources (i.e. seeing  $20 \div 5$  as 20 shared between 5) – a rather laborious process which can only be achieved by counting, and which becomes increasingly inefficient as both the divisor and the number to be divided by (the dividend) increase)

**These children are given little opportunity to use the grouping approach.**

**(i.e.  $20 \div 5$  means how many 5's are there in 20?) – far simpler and can quickly be achieved by counting in 5s to 20, something which most children in Y1 can do relatively easily.**

**Grouping in division can also be visualised extremely effectively using number lines and Numicon.** The only way to visualise sharing is through counting.

**Grouping, not sharing, is the inverse of multiplication.**

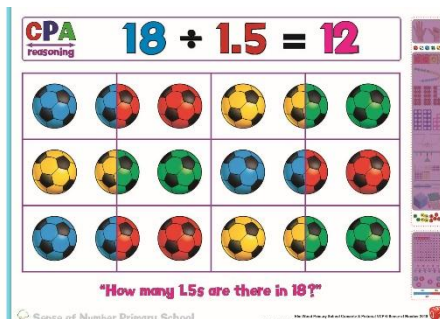
**Sharing is division as fractions.**



Once children have grouping as their first principle for division they can answer any simple calculation by counting in different steps (2s, 5s, 10s then 3s, 4s, 6s etc.). As soon as they learn their tables facts then they can answer immediately.

E.g. How much quicker can a child answer the calculations  $24 \div 2$ ,  $35 \div 5$  or  $70 \div 10$  using grouping? Children taught sharing would find it very difficult to even attempt these calculations.

Children who have sharing as their first principle tend to get confused in KS2 when the understanding moves towards 'how many times does one number 'go into' another'.

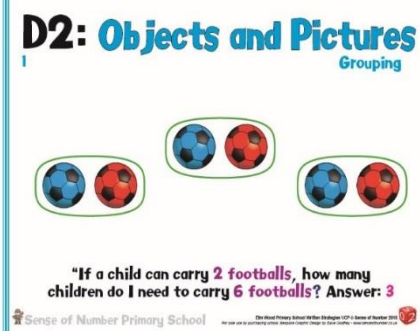
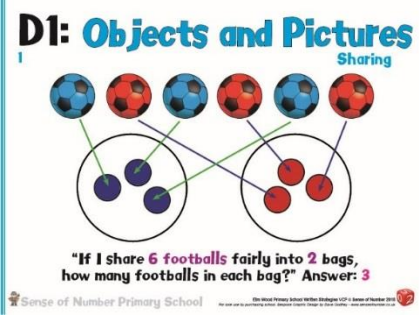

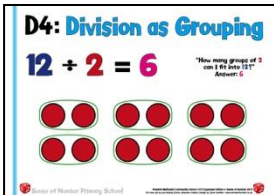
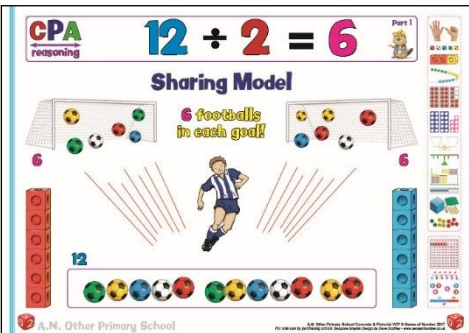
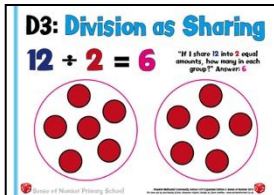
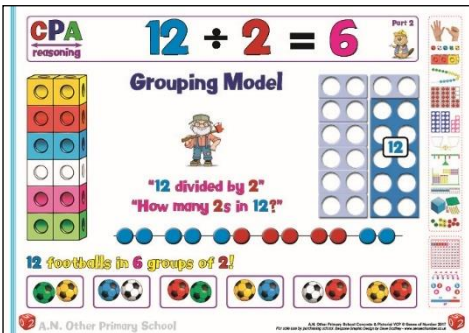


When children are taught grouping as their default method for simple division questions it means that they;

- secure understanding that the divisor is crucially important in the calculation
- can link to counting in equal steps on a number line
- have images to support understanding of what to do with remainders (Numicon)
- have a far more efficient method as the divisor increases
- have a much firmer basis on which to build KS2 division strategies

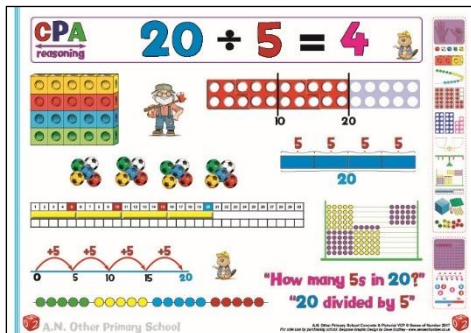
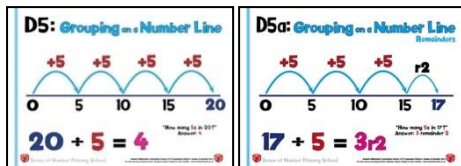
Consequently this policy is structured around the teaching of division as grouping, moving from counting up in different steps to learning tables facts and eventually progressing towards the mental chunking and 'bus stop' methods of written division in KS2.

Sharing is introduced as division in KS1, but is then taught mainly as part of the fractions curriculum, where the link between fractions and division is emphasised and maintained throughout KS2.

Stage 1	Concepts and Number Lines (pre-chunking)	
	Grouping	Sharing
FS	From EYFS onwards, children need to explore practically both <b>grouping</b> and <b>sharing</b> . Links can then be made in both KS1 and KS2 between sharing and fractions.	
Y1	Begin by giving children opportunities to use concrete objects, pictorial representations and arrays with the support of the teacher. Use the words 'sharing' and 'grouping' to identify the concepts involved. Identify the link between multiplication and division using the array image.	
		
		
Y2	Identify Grouping as the key model for division. Relate to knowledge of multiplication facts. Use the key vocabulary: '20 ÷ 5 means how many 5's can I fit into 20?'	Identify Sharing as the secondary model of division.
	 	 

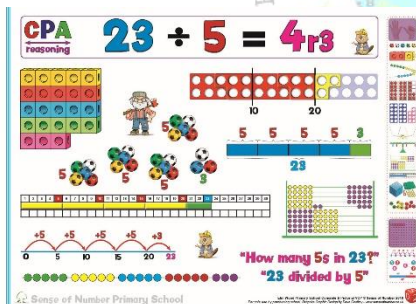
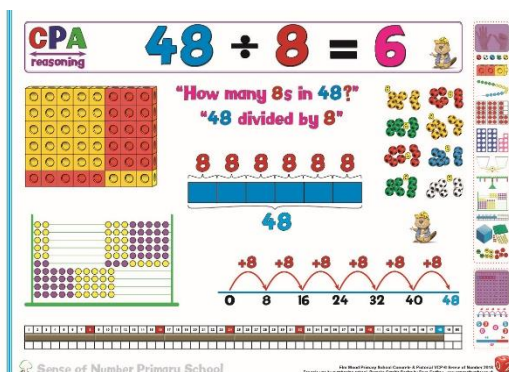


Counting on is the easiest route when using a number line to solve a division calculation.

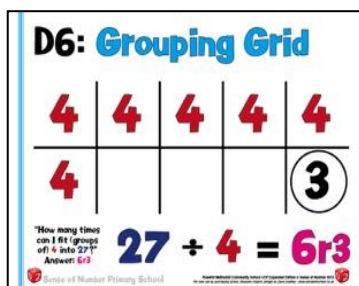


Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.

Y3



Continue to give children practical images for division by grouping: e.g. use PE mats and ask children to move into groups of 4. The remainder go into a hoop. Use Numicon shapes – how many 4 pieces can I fit into 27 (made of two tens and a seven piece).



Stage 2

Chunking & Standard Methods

Chunking

Standard Methods





# Find the Hunk & NNS Chunking

As previously encountered in Y2, developing an understanding of division with the number line is an excellent way of linking division to multiplication. It can show division both as repeated subtraction, but it is simpler to show division by counting forward to find how many times one number 'goes into' another.

## Y3

**D7: Chunking Jump**

4 x 10      4 x 8

+40      +32

0      40      72

**72 ÷ 4 = 18**

"How many 4s in 72?" Answer: 18

'Find the Hunk' is a mental strategy based on mental partitioning. For the example below, the Hunk is defined as being 10 times the divisor. i.e. the divisor is 4, so the Hunk will be 4 x 10 = 40. Both chunks are then divided by the divisor and then the groups totaled.

Where as 'Find the Hunk' is a mental strategy based on mental partitioning, the National Strategy chunking method is based on subtraction. Here 40 (4 x 10) is initially subtracted from the dividend. This strategy is somewhat confusing and the recommendation is to use Find the Hunk as the default strategy.

These slides introduce the Short Division (Bus Stop) method in Year 3. It is recommended that if children are introduced to this strategy in Year 3, it is only introduced at the end of Year 3 (ideally kept until Year 4) and that the key methods in Year 3 remain the use of Number Lines and the mental chunking method known as 'Find the Hunk' (see opposite)

When introducing Short Division formally, use dienes (Base 10) and make sure you introduce it using the **sharing model**. The calculation starts with, 'I have 7 tens, to share between 4 people. That's 1 each with 3 remaining. These three tens are exchanged into 30, ones. The 32 ones are now shared between 4 people – that's 8, ones each.'

**D6: Find the Hunk!**

The Hunk!      Chunk

**72 ÷ 4 = 18**

40      32

10      8

**10 + 8 = 18**

**(D11: Chunking)**

Additional

**18**

4 **72**

- 40 (4 x 10)

32

- 32 (4 x 8)

0

**72 ÷ 4 = 18**

**(D10: Short Division)**

Additional

**18**

**72 ÷ 4 = 18**

4 **72**

**CPA reasoning 72 ÷ 4 = 18**

10      8      40 + 32      Grouping Image!

4      40      32      10 + 8 = 18

1      2      3      4

10      10      10      10

1      1      1      1

4 **72**      4 **72**      4 **72**      4 **72**

1      1      1      1

18

4 **72**

Show the children examples of chunking where the quotient includes remainders.



**D6a: Find the Hunk!**  
Remainders

The Hunk!  $65 \div 4 = 16r1$

Chunk  $40 + 25 \div 4$

$10 + 6r1 = 16r1$

**D7a: Chunking Jump**  
Remainders

$4 \times 10$   $4 \times 6$  r1

$0 \quad +40 \quad +24 \quad 65$

$65 \div 4 = 16r1$

"How many 4s in 65?"

**D10a: Short Division**

$65 \div 4 = 16r1$

$4 \overline{) 65} \begin{matrix} 16 \\ r1 \end{matrix}$

**(D11: Chunking)**  
Abstract

$16r1$

$4 \overline{) 65}$

$-40 \quad (4 \times 10)$

$25$

$-24 \quad (4 \times 6)$

$1$

$65 \div 4 = 16r1$

**CPA Reasoning**  $65 \div 4 = 16r1$

10 6 r1  $40 + 25 \div 4$  Grouping Image!

1  $4 \overline{) 65}$

2  $4 \overline{) 65}$

3  $4 \overline{) 65}$  Sharing Image!

4  $4 \overline{) 65} \begin{matrix} 16 \\ r1 \end{matrix}$

'Mega Hunk' is the natural development of the 'Find the Hunk' strategy. Here Mega Hunk is defined as being multiple of 10 times the divisor. In the case below the divisor is 4, so the Hunk will be  $4 \times (10 \times 3) = 120$ . Again, both chunks are then divided by the divisor and then the groups totaled.

The National Strategy chunking method is also based on the multiples of 10 times the divisor. D11b slide is an expanded version of D11. Jottings can be made to spot the multiples of 10 times the divisor (e.g. 40, 80, 120 etc.). This strategy links to the **Grouping model**.

**Y4**

**D7: Mega Hunk!**

The Hunk!  $136 \div 4 = 34$

Chunk  $120 + 16 \div 4$

$30 + 4 = 34$

**D11: Chunking**

$34$

$4 \overline{) 136}$

$-120 \quad (4 \times 30)$

$16$

$-16 \quad (4 \times 4)$

$0$

$136 \div 4 = 34$

**D11b: Chunking**

$34$

$4 \overline{) 136}$

$-40 \quad (4 \times 10)$

$96$

$-40 \quad (4 \times 10)$

$56$

$-40 \quad (4 \times 10)$

$16$

$-16 \quad (4 \times 4)$

$0$

$136 \div 4 = 34$

**CPA Reasoning**  $136 \div 4 = 34$

30 4  $120 + 16 \div 4$   $30 + 4 = 34$

1  $4 \overline{) 136}$

2  $4 \overline{) 136}$

3  $4 \overline{) 136}$  It's a grouping image!

4  $4 \overline{) 136}$  It's a chunky group!

5  $4 \overline{) 136} \begin{matrix} 34 \end{matrix}$

**D10: Short Division**

$136 \div 4 = 34$

$4 \overline{) 136} \begin{matrix} 34 \end{matrix}$

**Y5**

Continue to use the Find the Hunk strategy whenever possible.



**D7d: Mega Hunk!** Remainders

$394 \div 6 = 65r4$

Mega Hunk!  $360 + 34 = 394$

$60 + 5r4 = 65r4$

Sense of Number Primary School

**D11c: Chunking** Remainders

$6 \overline{)394} = 65r4$

$-360$  ( $6 \times 60$ )

$34$

$-30$  ( $6 \times 5$ )

$4$

$394 \div 6 = 65r4$

Sense of Number Primary School

**D10c: Short Division**

$394 \div 6 = 65r4$

$6 \overline{)394} = 65r4$

Sense of Number Primary School

**CPA reasoning**  $394 \div 6 = 65r4$

1.  $6 \overline{)394}$

2.  $6 \overline{)394}$

3.  $6 \overline{)394}$

4.  $6 \overline{)394}$  Sharing Image!

5.  $6 \overline{)394}$   $65r4$

It's a sharing image!

Sense of Number Primary School

**D7e: Mega Hunk!**

$536 \div 4 = 134$

$400 + 120 + 16 = 536$

$100 + 30 + 4 = 134$

Sense of Number Primary School

**D11e: Chunking** Mega Chunk

$4 \overline{)536} = 134$

$-400$  ( $4 \times 100$ )

$36$

$-120$  ( $4 \times 30$ )

$16$

$-16$  ( $4 \times 4$ )

$0$

$536 \div 4 = 134$

Sense of Number Primary School

**D10e: Short Division**

$536 \div 4 = 134$

$4 \overline{)536} = 134$

Sense of Number Primary School

**CPA reasoning**  $536 \div 4 = 134$

1.  $4 \overline{)536}$

2.  $4 \overline{)536}$

3.  $4 \overline{)536}$

4.  $4 \overline{)536}$  Sharing Image!

5.  $4 \overline{)536}$

6.  $4 \overline{)536}$

Sense of Number Primary School

**D7g: Mega Hunk!**

$5978 \div 7 = 854$

$5600 + 350 + 28 = 5978$

$800 + 50 + 4 = 854$

Sense of Number Primary School

**D11e: Chunking** Mega Chunk

$7 \overline{)5978} = 854$

$-5600$  ( $7 \times 800$ )

$378$

$-350$  ( $7 \times 50$ )

$28$

$-28$  ( $7 \times 4$ )

$0$

$5978 \div 7 = 854$

Sense of Number Primary School

**D10e: Short Division**

$5978 \div 7 = 854$

$7 \overline{)5978} = 854$

Sense of Number Primary School

**Begin by subtracting several chunks, but then try to find the biggest chunks of the divisor that can be subtracted.**

Children should develop the ability to represent the quotient to include a straight forward remainder, but also as a decimal or fractional remainder.





**D7h: Mega Hunk!**

$$846 + 5 = 169r1$$

500 + 300 + 46  
 $\downarrow$     $\downarrow$     $\div 5$   
 100 + 60 + 9r1 = 169r1

**D11f: Chunking**   *Mega Chunk*

$$169r1$$

$$5 \overline{)846}$$

$$- 500 \quad (5 \times 100)$$

$$346$$

$$- 300 \quad (5 \times 60)$$

$$46$$

$$- 45 \quad (5 \times 9)$$

$$1$$

$846 + 5 = 169r1$

**D10f: Short Division**   *Different Remainders*

$$169.2$$

$$5 \overline{)846.0}$$

$$846 + 5$$

$$169r1$$

$$5 \overline{)846}$$

$$169 \frac{1}{5}$$

**Y6**

When introducing long division, it is often easier to find the quotient using the Mega Hunk strategy.

**D7i: Mega Hunk!**   *Simple Long Division*

$$480 + 15 = 32$$

Mega Hunk!   Chunk  
 450 + 30  
 $\downarrow$     $\div 15$   
 30 + 2 = 32

**D11g1: Chunking**   *Long Division*

$$32$$

$$15 \overline{)480}$$

$$- 450 \quad (15 \times 30)$$

$$30$$

$$- 30 \quad (15 \times 2)$$

$$0$$

$480 + 15 = 32$

**D11g2: Chunking**   *Long Division*

$$32$$

$$15 \overline{)480}$$

$$- 150 \quad (15 \times 10)$$

$$330$$

$$- 150 \quad (15 \times 10)$$

$$180$$

$$- 150 \quad (15 \times 10)$$

$$30$$

$$- 30 \quad (15 \times 2)$$

$$0$$

$480 + 15 = 32$

**D7k: Decimal Hunk!**

$$87.5 + 7 = 12.5$$

70 + 14 + 3.5  
 $\downarrow$     $\downarrow$     $\div 7$   
 10 + 2 + 0.5 = 12.5

**D7j: Decimal Hunk!**

$$18 + 1.5 = 12$$

The Hunk!   Chunk  
 15 + 3  
 $\downarrow$     $\div 1.5$   
 10 + 2 = 12

**D10i: Short Division**

$$87.5 + 7 = 12.5$$

$$12.5$$

$$7 \overline{)87.5}$$

**CPA reasoning**   **87.5 + 7 = 12.5**

$$70 + 14 + 3.5$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \div 7$$

$$10 + 2 + 0.5 = 12.5$$

10   2   0.5

**7**   **87.5**

**12.5**

*Sense of Number Primary School*

There are three different ways of calculating using Long Division: The Short Division method, the Traditional Method and the NNS Chunking method. The Traditional Long Division method ignores place value, and therefore is not as helpful as the Chunking Method, which now becomes the recommended strategy.



<p><b>D13: Long Division</b> Chunking Method</p> $\begin{array}{r} 26r21 \\ 37 \overline{) 983} \\ \underline{- 740} \quad (37 \times 20) \\ 243 \\ \underline{- 222} \quad (37 \times 6) \\ 21 \end{array}$ <p><math>983 \div 37 = 26r21</math></p>	<p><b>D13j: Long Division</b> Chunking Method</p> $\begin{array}{r} 26r21 \\ 37 \overline{) 983} \\ \underline{- 370} \quad (37 \times 10) \\ 613 \\ \underline{- 370} \quad (37 \times 10) \\ 243 \\ \underline{- 222} \quad (37 \times 6) \\ 21 \end{array}$ <p><math>983 \div 37 = 26r21</math></p>	<p><b>D12: Long Division</b> Short Division Method</p> $\begin{array}{r} 26r21 \\ 37 \overline{) 983} \end{array}$	<p><b>D14: Long Division</b> Traditional Method</p> $\begin{array}{r} 26r21 \\ 37 \overline{) 983} \\ \underline{- 74} \\ 243 \\ \underline{- 222} \\ 21 \end{array}$ <p><math>983 \div 37 = 26r21</math></p>
--	--	--	---

Written by Anthony Reddy, Dave Godfrey and Laurence Hicks – Sense of Number Maths Consultants

[www.senseofnumber.co.uk](http://www.senseofnumber.co.uk)

